Robust Output Feedback Trajectory Tracking Control of an Electrodynamic Planar Motion Stage for Precision Positioning

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Abstract— **The contribution addresses the problem of robust trajectory tracking control and disturbance rejection for a non contact, high-precision Lorentz force planar motion stage of linear DC brushless type. The control problem is accomplished by using a composite control law including a simple computed torque type controller together with an extended Luenberger type of state and disturbance observer also known as extended state or generalized proportional integral observer (GESO/GPIO). We provide stability proofs and estimation error bounds of the GESO/GPIO related to the choice of the eigenvalues of the observer estimation error dynamics. Finally, we assess the performance of the proposed controller on the basis of a recently developed motion stage model that comprises a realistic disturbance modeling and thus is capable of reflecting the major challenges for control.**

I. Introduction

Many applications in the field of nanotechnology and precision engineering such as scanning probe/force microscopy, lithography in semiconductor and microelectronic device fabrication as well as laser cutting in precision machining require submicron to nanometer positioning accuracy and high repeatability, while performing complex trajectory tracking tasks at high speed [1]. These tasks are normally undertaken in the plane over long traversing ranges.

For meeting these challenges and still increasing requirements non contact planar brushless type DC motors/Lorentz motors with three degrees of freedom are widely used as they provide nearly frictionless motion due to a levitated mover, e.g. through magnets or air bearings, high force density for dynamic positioning, and as they allow for linear motion over long travel ranges of several hundred millimeters [2]–[4].

However, these motors are known to exhibit (matched) external disturbances mainly originating from force ripples [5] that occur at the control input when subject to model uncertainties, i.e. commutation errors [6], [7], asymmetries and offsets in power amplifiers [6], magnetization effects of ferrous components in the traversing range (cogging-forces) [5], current ripple due to PWM-power-amplifiers [5], eddy current damping effects, spring force effects of air supply hoses and downhill-slope forces due to an imprecise or inaccurate stator adjustment, undesired coupling effects between motion axes and parameter variations caused by different ambient temperatures. These disturbances represent the major problems for position control.

To cope with these problems many approaches have been reported in the literature that range from simple PID control [3], PID feedback control with reference feedforward combined with a disturbance observer [2], PID control in combination with a sliding mode observer [4], to very complex composite control architectures as given in [5].

In joint research together with the planar motor manufacturer TETRA[1] we concentrate on developing a robust position controller for their newest precision positioner called LP^3S. It is desired that this particular controller is capable of trajectory tracking and (matched) disturbance rejection, while requiring a minimum of plant knowledge. One of the goals is that the disturbance rejection shall be accomplished by a disturbance estimator so as to obtain estimates of the total external disturbances and unmodeled dynamics for a compensation-based feedforward cancellation technique. For the trajectory tracking a simple computed torque controller is used that requires position as well as velocity measurements. However, usual planar motors such as the considered one lack velocity measurements, essential for this kind of control.

As a remedy, we propose to employ a combination of both a state and a disturbance observer. Both can be achieved by an extended Luenberger type of observer also known as generalized extended state observer (GESO) [8], [9] and generalized proportional integral observer (GPIO) [10], [11], respectively. Both observers are equivalent, where the origins of the GESO clearly lie in the active disturbance rejection control (ADRC) framework established in [12]. However, note that similar ideas date back to the 1970's [13]. Recently, the idea of GPIO has been placed into interesting, novel perspectives on the functionality and on valuable extensions that are relevant for practical implementations [11], [14].

The linear GESO/GPIO applies to a special class of nonlinear systems, where the basic idea is to lump all unknown linear/nonlinear dynamics such as unknown disturbances into a single parameter $\xi(t)$. This parameter is assumed to be m-times differentiable with respect to time, with derivatives bounded by some constant. When treating $\xi(t)$ and its derivatives up to order $m-1$ as additional system states, the nonlinear system transforms into a linear one which then is perturbed by $\xi^{(m)}(t)$. Subsequently, a standard observer is designed for the extended system, hence the name extended state observer. This approach is remarkable because of its simplicity, its excellent estimation performance and the fact that a minimum of plant information is required. Besides the conditions on boundedness and differentiability of $\xi(t)$, crucial for stability, only the plant's input gain has to be

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known. Therefore the GESO/GPIO may be considered an appropriate approach in the context of precision positioners and a wide variety of other applications.

To the best of the authors' knowledge, trajectory tracking control in combination with these observers has not yet been investigated in the context of precision positioning. In light of this, the paper aims at contributing a novel application to the GESO/GPIO community and a novel control approach to the precision engineering community. Furthermore, we provide a stability proof for the estimation error dynamics and also for the closed-loop system, and finally assess the performance of the overall controller with our recently developed simulation model of the considered motion stage [15].

The paper is structured as follows: After briefly introducing the operation principle of the planar motor and denotation in Section II, the basic modeling features are recalled from [15] in Section III. The main contribution, that is, the steps of controller design are presented in Section IV. The practical suitability of the advocated control approach is verified by means of simulation results, given in Section V. Section VI concludes the paper opening perspectives for future work.

II. Planar Motor Operation Principle

Fig. 1 shows a schematic of the planer motion stage. It essentially consists of a rigid, passive mover and an active stator base (non moving). The mover may be positioned in three degrees of freedom (DOF) in the plane: two translational x, y and one rotational ϕ. It is equipped with permanent magnet arrays (PMAs) on its back side and suspended by pressurized air employing aerostatic suspension bearings. This avoids dry friction and in particular stick-slip-effects such that smooth and nearly frictionless motion in the plane is possible. The stator part of the machine is a massive base unit made of granite which comprises four permanently fixed two-phase-coil-systems arranged in a crosswise manner. Such a PMA-coil-combination can be regarded as a linear motor (LM), as discussed on several publications, e.g. [2], [4], [16]. The electromagnetic forces/torques (F_x, F_y, M_z) propelling the mover along its motion axes (x, y, ϕ) are produced by the LMs (see Fig. 2). As a consequence of Lorentz' Law these forces/torques are proportional to the coil-currents applied, acting perpendicular wrt. the direction of current flow as well as perpendicular to the magnetic field direction.

To control the phase currents, considered as control inputs, four PWM two-phase digital current amplifiers (DCAs) are employed as servo-amplifiers for the resp. two-phase-coil-systems (cf. Fig. 2). We call a DCA-LM-combination a linear servomotor (LSM). Note that the architecture described above is widely used and similar to other motor concepts reported in the literature (see e.g. [2]–[4], [16], [17]).

For clarity of notation we denote the elements of the four LSMs that produce thrust forces in the x- and y-directions by X_i and Y_i, respectively, where $i = 1, 2$ serves as system index. Note that the motors (X_1, X_2) such as (Y_1, Y_2) oppose each other. Usually the LM pair (X_1, X_2) is driven simultaneously in order to achieve symmetric movement in the direction of x. The same applies for the LMs (Y_1, Y_2). The phase index $j = 1, 2$ in X_{ij} and Y_{ij} is to represent the j-th phase of the i-th LM, coil-system or DCA for the x- and y-directions. Accordingly, the eight real-valued scalar coil phase currents are expressed by $i_{x_{ij}}(t)$ and $i_{y_{ij}}(t)$. They are measured and driven to their reference trajectories $i^r_{x_{ij}}(t)$ and $i^r_{y_{ij}}(t)$ by the DCAs.

In order to capture the mover's motion, an earth-fixed (inertial) coordinate frame (i-frame) (x, y, z) located at the center of the stator surface and a body-fixed frame (b-frame) (x^b, y^b, z^b) coincident with the center of gravity (CG) and principle axes of rotation is used (cf. Fig. 1). The position coordinates x and y of the mover's CG and its yaw angle ϕ are measured with an optical, incremental sensor (with a position resolution of 2.44 nm) underneath the mover.

Fig. 2 shows a typical control task, where a reference $\tau^r(t) = \left(F^r_x(t), F^r_y(t), M^r_z(t)\right)^{\mathrm{T}}$ is specified for the generalized forces $\tau(t) = (F_x(t), F_y(t), M_z(t))^{\mathrm{T}}$ on the mover such that its position $q(t) = (x(t), y(t), \phi(t))^{\mathrm{T}}$ tracks a desired reference trajectory $q^r(t) = (x^r(t), y^r(t), \phi^r(t))^{\mathrm{T}}$. A motor commutation law maps the desired reference $\tau^r(t)$ to the corresponding phase current reference $i^r(t) = \left(i^r_{x_{11}}(t), i^r_{x_{12}}(t), \ldots, i^r_{y_{21}}(t), i^r_{y_{22}}(t)\right)^{\mathrm{T}}$. The current control loop of the DCAs assures $i(t) \to i^r(t)$ for the measured current $i(t) = (i_{x_{11}}(t), i_{x_{12}}(t), \ldots, i_{y_{21}}(t), i_{y_{22}}(t))^{\mathrm{T}} \in \mathbb{R}^8$. Through the electromagnetic coupling of forces, from $i(t) \to i^r(t)$ we have $\tau(t) \to \tau^r(t)$, thus $q(t) \to q^r(t)$, as desired.

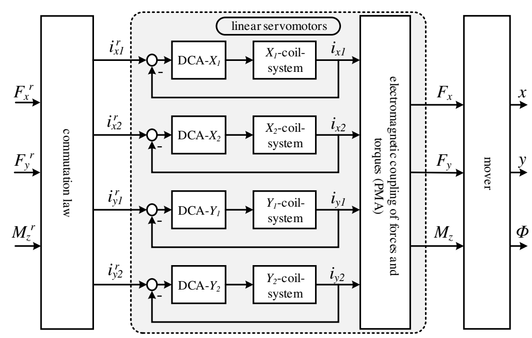

Fig. 2. Signal flow diagram of the motion stage, where the quantities $i_{x_i} = (i_{x_{i1}}, i_{x_{i2}})^{\mathrm{T}}$ and $i_{y_i} = (i_{y_{i1}}, i_{y_{i2}})^{\mathrm{T}}$ while $i = 1, 2$ are vectors of the corresponding phase currents. The same applies to $i^r_{x_i}$ such as $i^r_{y_i}$.

III. Modeling

For the analysis, design, and assessment of controllers it is important to have at one's disposal a model that captures the most relevant phenomena reflecting the main challenges for control. To this end and with regard to Fig. 2, we briefly summarize the planar motion stage model that has been subject of our recent investigations [15].

The overall motion stage model may be expressed by a perturbed Euler-Lagrange-system of the form

$$M\ddot{q}(t) + D\dot{q}(t) = \epsilon\tau^r(t) + d(q, \dot{q}), \tag{1}$$

with mass matrix $M \in \mathbb{R}^{3\times3} > 0$ and damping matrix $D \in \mathbb{R}^{3\times3} > 0$. Quantity $\epsilon > 0$ models input gain uncertainties, $\tau^r(t) \in \mathbb{R}^3$ is the control input and $d(q, \dot{q})$ denotes periodic,

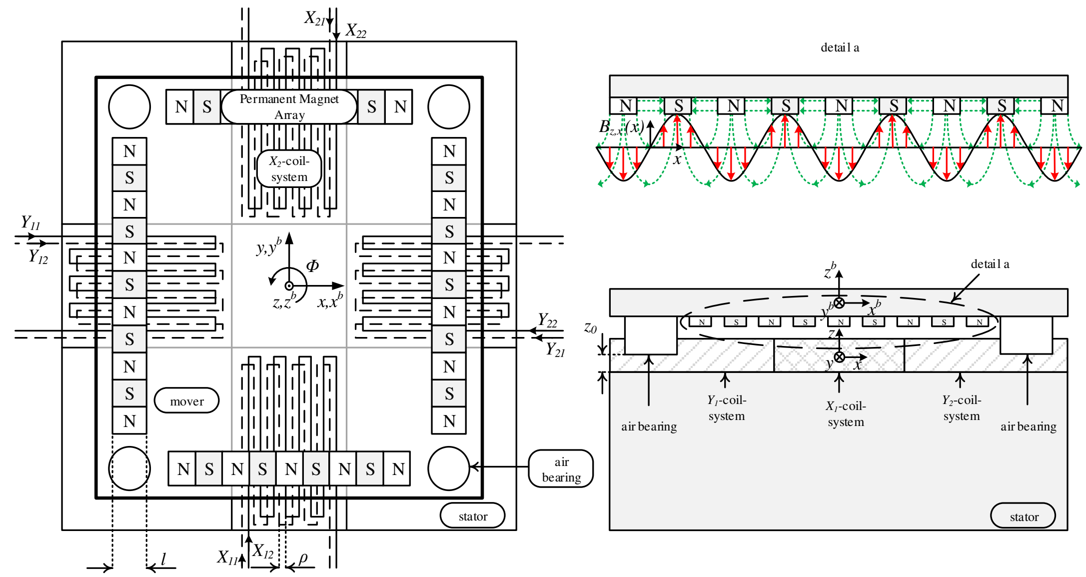

Fig. 1. Top view of motion stage (left), side view of X_1-permanent magnet array (PMA) (top right) with magnetic field lines (green), vertical vector components of the flux density (red) and sinusoidal magnitude of the vertical vector components (black). Side view of motion stage (bottom right).

position and velocity dependent force ripples originating from commutation errors, offsets/asymmetries in the actuation stages (LSMs) and external disturbances, see [15].

For precise trajectory tracking, the controller has to reject the input disturbances $d(q, \dot{q})$ which is challenging since the frequency of the periodic force ripples scales with the mover velocity. A detailed discussion on this can be found in [6]. Consequently, all the disturbances accumulated in $d(q, \dot{q})$ appear at the control input as matched uncertainties, where the influence on the overall system turns out strong.

IV. Controller Design

A. Problem Statement

We consider the perturbed mechanical system (1) with only M known. The disturbance $d(q, \dot{q})$, input gain uncertainty $\epsilon > 0$ and D are all unknown. Moreover, only $q(t)$ is measurable.

The underlying control problem may be stated as follows:

Devise a uniformly bounded and sufficiently smooth control law for τ^r such that $q(t)$ tracks a desired sufficiently smooth reference trajectory $q^r(t)$ while attenuating unknown disturbances as well as the unmodeled dynamics.

B. Nominal Control Law

In the nominal case assume $\epsilon = 1$ and M, $d(q, \dot{q})$ and D perfectly known. Moreover, position $q(t)$ and velocity $\dot{q}(t)$ shall be measurable. The nominal tracking control law may then simply be chosen as

$$\begin{aligned} \tau^r(t) &= M\, v(t) + D\, \dot{q}(t) - d(q, \dot{q}) \\ v(t) &= \ddot{q}^r(t) - k_d(\dot{q}(t) - \dot{q}^r(t)) - k_p(q(t) - q^r(t)) \end{aligned} \tag{2}$$

which essentially is a standard PD-controller with compensation and reference feedforward of computed torque control style.

Introducing the tracking error $e(t) = q(t) - q^r(t)$ it is well-known that the closed-loop error dynamics

$$\ddot{e}(t) + k_d\, \dot{e}(t) + k_p\, e(t) = 0 \tag{3}$$

is globally asymptotically stable if the gains $k_p, k_d \in \mathbb{R}^{3\times 3}$ are chosen both symmetric and positive definite.

In the perturbed case, we propose to employ an extended Luenberger type of observer, a so-called generalized extended state observer (GESO) [8], [9] also known as generalized proportional integral observer (GPIO) [10], [11]. This observer encompasses both a state observer and a disturbance observer, thus, provides estimates of the unknown state $\dot{q}$, and unknown perturbations that occur either through external disturbances, model uncertainties or unmodeled dynamics, i.e. through the unknown terms $D\, \dot{q}(t)$ and $d(q, \dot{q})$. The estimates may then be used in the afore-presented controller for the online disturbance rejection and trajectory tracking. The advocated approach is much related to the well known active disturbance rejection control (ADRC) framework [12] shown to be powerful in many applications such as flight control [18], web tension regulation [19], and robotics [20] among others.

C. GESO/GPIO Analysis

Both GESO and GPIO are similar, generalized/modified linear gain versions of the original extended state observer (ESO) [12] being a fundamental part of the (ADRC) framework. The basic idea of the ESO concept is to lump all unknowns in one variable $\xi(t)$, the so-called generalized/total disturbance [12], and treat it as an extended system state. Originally, the ESO provided estimates of all states and an estimate of $\xi(t)$ only, while generalizations furthermore provide estimates of $\xi(t)$ up to its $(m-1)$-th time-derivative, i.e. they incorporate an internal disturbance model in terms of an $(m-1)$-degree Taylor polynomial (see [10], [11] and references therein).

Subsequently, we apply the GESO/GPIO on the perturbed model in the line of thought as in [8], [10], [11]. For this purpose, we reformulate (1) as per

$$\ddot{q}(t) = \underbrace{M^{-1}(d(q,\dot{q}) - D\,\dot{q}(t))}_{=:f(q,\dot{q},d)} + \underbrace{\epsilon M^{-1}}_{=:\beta}\tau^r(t), \tag{4}$$

where $\dot{q}$, d, and the vector-valued function f are considered unknown. We assume that function $f(\cdot)$ is sufficiently smooth such that a unique solution $q = q(t)$ exists for every given set of initial conditions $q(t_0) = q_0$. Moreover, $q(t)$ is assumed to be measured. Parameter ϵ shall only be used for studying the closed-loop performance when affected by uncertain input gains. Hence, for the time being let $\epsilon = 1$ while M is assumed to be perfectly known. According to [11] we proceed by defining $\xi : t \mapsto f(q(t), \dot{q}(t), d(q(t), \dot{q}(t)))$ that maps time t via $q(t)$ and $\dot{q}(t)$ to the vector value of f. Intuitively, signal $\xi(t) \in \mathbb{R}^3$ captures all information of the unknown function f at every time instant t. We further assume that all time derivatives up to the m-th time derivative, $\xi^{(m)}(t)$, exist and are bounded by some positive constants κ_l [10], that is

$$\sup_t \|\xi^{(l)}(t)\| \leq \kappa_l, \quad 0 \leq l \leq m \tag{5}$$

where $\|\cdot\|$ denotes some vector norm[2]. We may now treat the signal $\xi(t)$ up to its $(m-1)$-th time-derivative as additional system states. Denoting the original system states as $\eta_1 = q$, $\eta_2 = \dot{q}$ and the additional m states marking the disturbance as $\eta_3 = \xi$, $\eta_4 = \dot{\xi}$, $\ldots$, $\eta_{m+2} = \xi^{(m-1)}$, hence $\eta \in \mathbb{R}^{3(m+2)}$, the extended system dynamics of (4) reads

$$\begin{aligned} \dot{\eta}(t) &= A_e\,\eta(t) + B_e\,\tau^r(t) + B_\xi\,\xi^{(m)}(t) \\ y(t) &= C_e\,\eta(t), \end{aligned} \tag{6}$$

with $y(t) \in \mathbb{R}^3$ the measured output and

$$A_e = \begin{pmatrix} 0 & I & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \vdots & & & \ddots & I \\ 0 & \cdots & \cdots & \cdots & 0 \end{pmatrix},\ B_e = \begin{pmatrix} 0 \\ \beta \\ 0 \\ \vdots \\ 0 \end{pmatrix},\ B_\xi = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ I \end{pmatrix}$$

$$C_e = \begin{pmatrix} I & 0 & \cdots & 0 \end{pmatrix}$$

of appropriate size. I and 0 are 3×3 identity and zero matrix, respectively. It is easy to verify that the pair (A_e, C_e) is observable. Therefore, the unknown states $\eta_2, \ldots, \eta_{m+2}$ of the extended state space representation (6) may be reconstructed by a high gain observer of Luenberger type, i.e.

$$\begin{aligned} \dot{\hat{\eta}}(t) &= A_e\,\hat{\eta}(t) + B_e\,\tau^r(t) + L\,(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_e\,\hat{\eta}(t). \end{aligned} \tag{7}$$

Here, $\hat{\eta}(t)$ denotes the estimate of $\eta(t)$ and $L \in \mathbb{R}^{3(m+2)\times3}$ is the observer gain. In view of assumption (5) and by observability, the proposed observer may globally asymptotically force the estimation error $\tilde{\eta} = \eta - \hat{\eta}$ in a small neighborhood around zero, where it remains bounded for all times. Please note that the number m of additional system states associated with $\xi(t), \ldots, \xi^{(m-1)}(t)$ is directly related with the degree of the internal Taylor time-polynomial disturbance model. Thereby, a higher polynomial degree allows for a better approximation of complex disturbances. However, finding a suitable number m is consequently a trade-off between good disturbance approximation and low observer system order.

[2]The boundedness assumption on the time derivatives up to the $(m-1)$-th ensures the boundedness of the observer states, used for feedback control (bounded control signals). If state/disturbance estimation (without control) is the task then it may be sufficient to assume boundedness of $\xi^{(m)}$, only.

For the sake of completeness and as a complaisant correction of an error in [10] we proof the stability result.

Proof: Subtracting (7) from (6) yields the error dynamics $\dot{\tilde{\eta}} = \tilde{A}_e\tilde{\eta} + B_\xi\xi^{(m)}$ where $\tilde{A}_e = A_e - LC_e$ is rendered stable with a proper selection of L, e.g. by eigenvalue assignment.

Take the Lyapunov function candidate

$$V(\tilde{\eta}) = \frac{1}{2}\tilde{\eta}^{\mathrm{T}}P\tilde{\eta}, \tag{8}$$

with $P^{\mathrm{T}} = P > 0$ and compute

$$\begin{aligned} \dot{V} &= \frac{1}{2}\left(\dot{\tilde{\eta}}^{\mathrm{T}}P\tilde{\eta} + \tilde{\eta}^{\mathrm{T}}P\dot{\tilde{\eta}}\right) \\ &= \frac{1}{2}\left(\tilde{\eta}^{\mathrm{T}}\left(\tilde{A}_e^{\mathrm{T}}P + P\tilde{A}_e\right)\tilde{\eta} + 2\tilde{\eta}^{\mathrm{T}}PB_\xi\xi^{(m)}\right) \\ &= -\frac{1}{2}\tilde{\eta}^{\mathrm{T}}Q\tilde{\eta} + \tilde{\eta}^{\mathrm{T}}PB_\xi\xi^{(m)}, \end{aligned}$$

where $Q^{\mathrm{T}} = Q > 0$ and due to $\tilde{A}_e$ stable, P is the unique solution of the Lyapunov equation $\tilde{A}_e^{\mathrm{T}}P + P\tilde{A}_e = -Q$. In view of (5) we obtain

$$\begin{aligned} \dot{V} &\leq -\frac{1}{2}\lambda_{\min}(Q)\|\tilde{\eta}\|^2 + \tilde{\eta}^{\mathrm{T}}PB_\xi\kappa_m \\ &\leq -\frac{1}{2}\lambda_{\min}(Q)\|\tilde{\eta}\|^2 + \lambda_{\max}(P)\|\tilde{\eta}\|\,\|B_\xi\|\,\kappa_m. \end{aligned}$$

Therefore, imposing $\dot{V} \leq 0$ means that

$$\|\tilde{\eta}\| \geq \frac{2\,\lambda_{\max}(P)\,\|B_\xi\|\,\kappa_m}{\lambda_{\min}(Q)} \geq \frac{\|B_\xi\|\,\kappa_m}{\alpha}, \tag{9}$$

where the latter inequality may be drawn from [21], for simplicity. Therein, $\alpha = \min_i |\mathrm{Re}(\lambda_i(\tilde{A}_e))|$ is the absolute real part of the right most eigenvalue of $\tilde{A}_e$. Hence all solutions starting outside of this sphere will eventually converge towards its interior which can be made arbitrarily small increasing the observer gain L. ∎

In general, the required high observer gains might be problematic when the measurements are subject to high frequency noise. In practice, this noise amplification may be reduced when integrating the measured output variable $y(t)$ and feeding the low-pass filtered variable to the observer, while adapting the extended state space representation of the observer by the number of additional integrations. This idea has been published by [14] and is known as GPI$_+$ or GPI with integral injection. Thus a practical implementation of these observers is definitely possible as can be seen by the number of successful applications (see [10], [11], [14], [22] and references therein).

D. Observer-based Trajectory Tracking Control Law

With the properties of the GESO/GPIO explored above we are in the position to state the observer-based trajectory tracking control law

$$\tau^r(t) = M\left(v(t) - \hat{\xi}(t)\right) \tag{10}$$
$$v(t) = \ddot{q}^r(t) - k_d(\dot{\hat{q}}(t) - \dot{q}^r(t)) - k_p(q(t) - q^r(t))$$

making use of the state and disturbance estimates provided by the observer proposed in (7).

Closing the loop of (4) with control law (10) the tracking error dynamics can then be expressed in terms of tracking error $e = q - q^r$, state estimation error $\dot{\tilde{q}} = \dot{q} - \dot{\hat{q}}$, and disturbance estimation error $\tilde{\xi} = \xi - \hat{\xi}$, namely

$$\ddot{e}(t) + k_d\,\dot{e}(t) + k_p\,e(t) = k_d\,\dot{\tilde{q}}(t) + \tilde{\xi}(t). \tag{11}$$

The error dynamics is driven by $k_d\,\dot{\tilde{q}}(t) + \tilde{\xi}(t)$. Due to the properties of the GESO/GPIO we know that the estimation errors $\tilde{\eta}_2 = \dot{\tilde{q}}$ and $\tilde{\eta}_3 = \tilde{\xi}$ are bounded and may be kept in a small neighborhood around the origin of the estimation error space. Hence choosing the controller gains k_p, k_d symmetric and positive definite, see Section IV-B, the tracking error $e(t)$ will consequently globally asymptotically converge to a small neighborhood around the origin of the tracking error space, as desired.

V. Simulation Results

In order to assess the performance of the proposed trajectory tracking and disturbance rejecting control law, simulations have been carried out using the perturbed motion stage model (1) of system class (4). Note that this model has been accurately identified at the test rig such that the simulation environment realistically reflects the main control challenges (see [15] for a comparison between plant measurements and simulation results). However, limitations as well as resolutions of position sensors have not yet been considered within the simulation environment. Fig. 5 pictures simulation results performed in closed loop (y-axis) for the case where the input gain β is perfectly known and $\epsilon = 1$. Evidently, the commanded reference trajectories for position, velocity and acceleration are tracked well, also the estimates of velocity $\dot{y}(t)$ and disturbance $f_y(t)$. The lower plots of Fig. 5 show that the transient of the estimation error exhibits large initial peaking, typical for high gain observers. If reducing the initial peaking is an issue, *smooth clutching* techniques discussed in [11] may be applied.

Fig. 3 illustrates the tracking error in the y-direction when the system is affected by uncertainty in the gain ϵ. It reveals that the closed-loop performance is quite sensitive to variations of ϵ, while an underestimation in the input gain meaning $M < \epsilon M$ is less critical than an overestimation, i.e. $M > \epsilon M$. Intuitively, the latter involves greater control inputs which results in a more sensitive closed-loop behavior. Thus, the proposed algorithm requires an accurate estimate of the input gain which in the context of a planar motor is given by the ratio of motor constant and mover mass. Indeed, these parameters are known well since they represent essential design parameters of a motion stage. However, recall that knowledge about damping coefficients, coupling of states due to disturbance effects, etc., is not required.

In order to assess the effectiveness of disturbance rejection, in Fig. 4 we compare the tracking error obtained by the proposed controller with and without the disturbance cancellation term $\hat{\xi}$. Thereby we keep the controller/observer parameters as in the simulation studies before (see caption of Fig. 5). Note that omitting $\hat{\xi}$ in the proposed control law leads to a conventional PD-controller with acceleration feedforward. Apparently, the proposed controller outperforms the conventional one. It almost entirely rejects the occurring disturbances, whilst the PD-controller is only capable of slightly attenuating them. This result is not surprising since the PD-controller compared to the proposed one lacks integral terms incorporating an internal model of the disturbance.

VI. Conclusion and Future Work

We investigated the application of a robust output feedback tracking controller with disturbance rejection in the context of precision positioning. The core of the controller is an extended state observer that serves to estimate both unknown state variables and unknown disturbances including unmodeled dynamics to the end of control and disturbance rejection. Stability is proven for the estimation error dynamics and the closed-loop system, while providing a bound in terms of the eigenvalues of the estimation error dynamic matrix $\tilde{A}_e$. Remarkable properties of the proposed control approach are its simplicity, the minimum need for plant and disturbance information that makes the approach applicable to a wide variety of systems, and the excellent performance in estimating the unknowns. The latter was shown by simulation studies on an accurate precision positioning motion stage model reflecting the main challenges for control, which had been subject of our recent investigations [15]. A future concern will be the (discrete-time) implementation of the presented control scheme on the experimental motion stage platform.

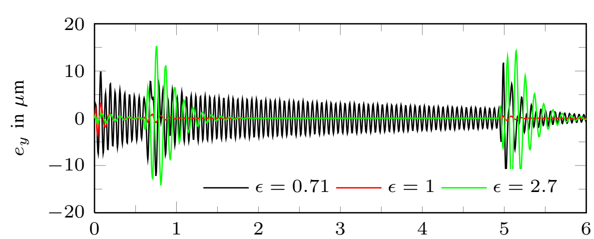

Fig. 3. Position tracking error e_y for different input gain uncertainty ϵ.

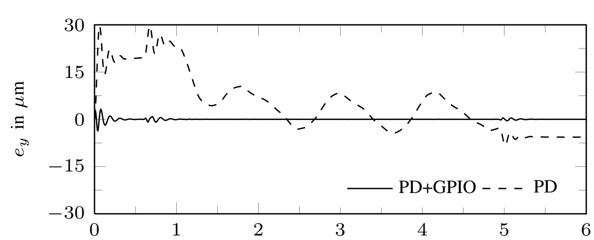

Fig. 4. Comparison of the proposed algorithm with disturbance rejection and a conventional PD-controller with acceleration feedforward.

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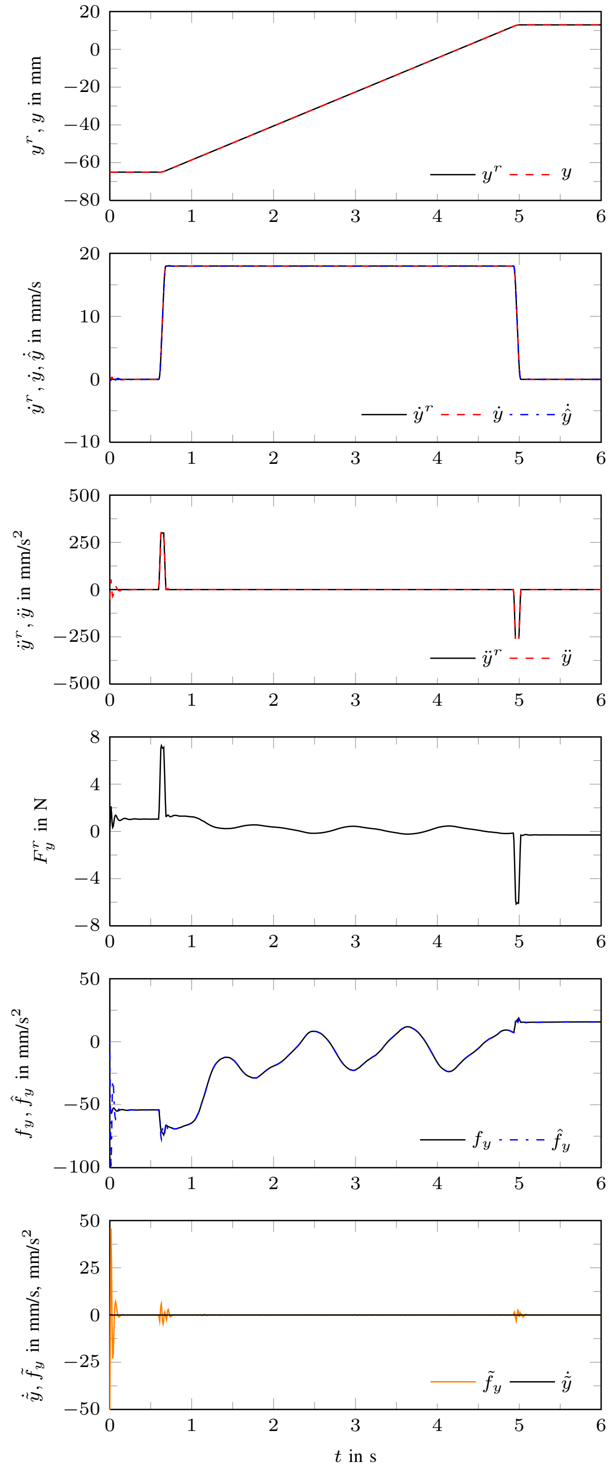

Fig. 5. Simulation of the perturbed system (1) controlled by the proposed controller (10). The subplots show (from top to bottom) position, velocity, acceleration, control input, disturbance f_y and estimation error $\tilde{f}_y$ in the direction of y together with their commanded references (superscript r) and their estimates (hat), respectively. The controller parameters are $k_p = \omega_0^2 I$ and $k_d = 2\zeta\omega_0 I$, whereupon $\zeta = 0.7$ and $\omega_0 = 4\pi$ such that the error dynamics behaves as a second order mass-spring-damper-model. The observer poles were all placed at $\lambda_k = -120$ with $k = 1, \dots, 3(m+2)$. Thereby in order to obtain a good disturbance approximation, while keeping the observer order low we chose $m = 5$. Moreover $\epsilon = 1$. The simulation results illustrate that despite of the simplicity of the proposed control algorithm the trajectory tracking, state and disturbance estimation as well as disturbance rejection performs remarkably well.