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Observability Analysis and Nonlinear Observer Design for a Turbocharger in a Diesel Engine

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Abstract— **In this contribution we consider the observer design for an exhaust-gas turbocharger of a diesel combustion engine based on a nonlinear physical model. We consider two different observability properties and present a numerical as well as an analytical observability analysis of the nonlinear system. Based on that we derive explicit constraints for the system state that guarantee observability. Furthermore, we design a High-Gain observer and an Extended Kalman filter using a reduced-order and a full-order system, respectively. Since the nonlinear model is non-Lipschitz we employ numerical linearization techniques for the implementation of the Extended Kalman filter. Both observers are implemented and tested using data from an engine test-rig.**

I. INTRODUCTION

Nowadays automotive engines have to be increasingly efficient while reducing emissions at the same time. One way to achieve a better power utilization is the use of turbocharging. Fig. 1 shows the structure of such motor air path system containing an exhaust-gas turbocharger a compressor rises the fresh air pressure and thus the capability of carrying oxygen to the combustion and its power output. The exhaust gas flow is then passing a turbine which uses the remaining energy to drive the compressor. The turbocharger considered in this paper exhibits a so-called variable turbine-geometry (VTG) that allows to adjust the power that drives the turbine. This control-input allows to adjust the boost-pressure p_{IM} to some desired value, whenever the exhaust-gas energy is large enough.

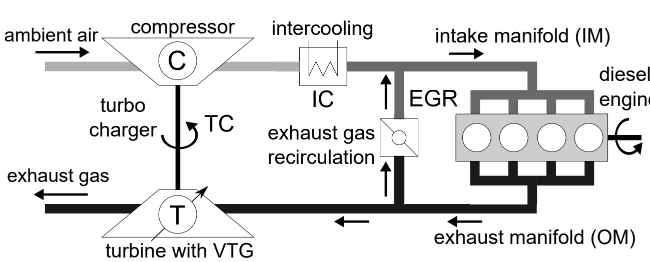

Fig. 1. Structure of diesel engine air path system with turbocharger.

The boost-pressure control task is challenged by several issues. The process dynamics are highly nonlinear and depend strongly on external variables such as the engine operating point or the exhaust-gas recirculation rate (EGR). Many parameters and characteristics are unknown and difficult to identify such that available models suffer from large uncertainties. Important signals are not available for the controller since appropriate sensors are costly.

In the recent past the observer design for turbocharger systems have gained some attention [1], [2], [3]. In [2] a nonlinear Luenberger observer is presented for the system of an SI engine without EGR. Diesel engines with EGR are considered in [1], [3] using sliding-mode based approaches. [1] investigates the estimate of the air mass fractions and exhaust manifold pressure. In [3] a reduced-order model is considered and the EGR mass flow is treated as a known input. To the best of the authors' knowledge an observability analysis for this system is not yet provided in the literature.

The focus of this contribution is the estimation of the turbocharger rotation speed using only measurements of the boost-pressure (intake manifold) and a parameterized model. We consider both, a sixth-order model as well as a reduced third-order model. Further we compare two different methods to the observer-design: The High-Gain observer is in particular suitable as the system's nonlinearity may be included into the observer design and no (possibly nonlinear) transformation into a normal form is needed. Also, this class of observers is simple to tune and exhibits a large robustness against modelling uncertainties. The second approach is the classical Extended Kalman filter, which represents the industry benchmark. Essentially both observer approaches are not applicable to systems with non-Lipschitz dynamics. However, for each case we propose techniques that allow for a successful implementation and testing of the algorithms.

To investigate observability of the system we consider two concepts of observability available for nonlinear systems, namely, weak observability and uniform observability. Local weak observability is a basic condition for the design of observers, [4]. It turns out that the system is not globally weakly observable, but exhibits a considerable subset of states that are not observable. The uniform observability as introduced in [5] is a stronger concept and allows for the stable design of High-Gain observers.

The paper is structured as follows. In the following section the nonlinear model is presented which combines physical considerations and experimental identification techniques to obtain a sufficiently accurate system representation. Section III presents the observability analysis of the nonlinear system using the observability rank condition analytically and for more extensive calculations numerically. In Sections IV and V we present the High-Gain observer and Extended Kalman filter design, resp. Both observers are implemented and evaluated using data from an engine test-rig. Results are presented in Section VI.

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II. MODELING

We use a fully parameterized nonlinear model of the diesel engine air path system with turbocharger which is based on [6]. All variables and parameters are listed in the table below.

$$\dot{p}_{\mathrm{IM}} = \frac{R\,T_{\mathrm{IM}}}{V_{\mathrm{IM}}}\left(W_{\mathrm{C}} + W_{\mathrm{EGR}} - W_{\mathrm{cyl}}^{\mathrm{in}}\right), \tag{1}$$

$$\dot{n}_{\mathrm{TC}} = \frac{1}{J}\,\frac{\eta_{\mathrm{m}} P_{\mathrm{T}} - P_{\mathrm{C}}}{n_{\mathrm{TC}}}, \tag{2}$$

$$\dot{p}_{\mathrm{OM}} = \frac{R\,T_{\mathrm{UT}}}{V_{\mathrm{EM}}}\left(W_{\mathrm{cyl}}^{\mathrm{out}} - W_{\mathrm{EGR}} - W_{\mathrm{T}}\right), \tag{3}$$

$$\dot{T}_{\mathrm{UT}} = \frac{R\,T_{\mathrm{UT}}}{V_{\mathrm{EM}}\,p_{\mathrm{OM}}}\kappa\Big(T_{\mathrm{OM}} W_{\mathrm{cyl}}^{\mathrm{out}} - T_{\mathrm{UT}}(W_{\mathrm{EGR}} + W_{\mathrm{T}})\Big) - \frac{T_{\mathrm{UT}}}{p_{\mathrm{OM}}}\dot{p}_{\mathrm{OM}}, \tag{4}$$

$$\dot{u}_{\mathrm{VTG}} = \frac{1}{\tau_{\mathrm{VTG}}}\left(u_{\mathrm{VTG}}^{\mathrm{ref}} - u_{\mathrm{VTG}}\right), \tag{5}$$

$$\dot{u}_{\mathrm{EGR}} = \frac{1}{\tau_{\mathrm{EGR}}}\left(u_{\mathrm{EGR}}^{\mathrm{ref}} - u_{\mathrm{EGR}}\right). \tag{6}$$

The system is derived from thermodynamic mass and power balances, see e. g. [7] for details. The actuators are modelled by linear first-order dynamics. For brevity the arguments of some functions will be omitted, whenever ambiguity can be ruled out. Following the analysis of the turbine mass-flow in [8], the flow rates as well as the compressor and turbine powers are given by:

$$W_{\mathrm{cyl}}^{\mathrm{in}} = k_1\, p_{\mathrm{IM}},$$

$$W_{\mathrm{cyl}}^{\mathrm{out}} = W_{\mathrm{cyl}}^{\mathrm{in}} + k_2,$$

$$W_{\mathrm{C}} = k_3\,\Phi(p_{\mathrm{IM}}, n_{\mathrm{TC}})\, n_{\mathrm{TC}},$$

$$W_{\mathrm{T}} = k_4\,\Psi(p_{\mathrm{OM}}, u_{\mathrm{VTG}})\, p_{\mathrm{OM}}\, A_{\mathrm{VTG}}(u_{\mathrm{VTG}}),$$

$$W_{\mathrm{EGR}} = k_5\, p_{\mathrm{OM}}\sqrt{2\frac{p_{\mathrm{IM}}}{p_{\mathrm{OM}}}\left(1 - \frac{p_{\mathrm{IM}}}{p_{\mathrm{OM}}}\right)}A_{\mathrm{EGR}}(u_{\mathrm{EGR}}),$$

$$P_{\mathrm{C}} = \frac{k_{\mathrm{C}} W_{\mathrm{C}}}{\eta_{\mathrm{C}}(p_{\mathrm{IM}}, W_{\mathrm{C}})}\left(\left(\frac{p_{\mathrm{IM}}}{p_{\mathrm{amb}}}\right)^{\frac{\kappa-1}{\kappa}} - 1\right),$$

$$P_{\mathrm{T}} = k_{\mathrm{T}} W_{\mathrm{T}}\, T_{\mathrm{UT}}\left(1 - \left(\frac{p_{\mathrm{amb}}}{p_{\mathrm{OM}}}\right)^{\frac{\kappa-1}{\kappa}}\right)\eta_{\mathrm{T}}(u_{\mathrm{VTG}}),$$

with positive known and constant parameters $k_1, k_2, k_3, k_4, k_5, k_{\mathrm{C}}, k_{\mathrm{T}}$. In this study k_1 and k_2 depend on the constant motor operating point. In fact k_2 represents the fuel injection and k_1 is proportional to the motor rotational speed. The pressure p_{IM} can be directly measured and forms the system output. For the functions $A_{\mathrm{VTG}}(u_{\mathrm{VTG}})$, $A_{\mathrm{EGR}}(u_{\mathrm{EGR}})$, $\eta_{\mathrm{C}}(p_{\mathrm{IM}}, W_{\mathrm{C}})$, $\eta_{\mathrm{T}}(u_{\mathrm{VTG}})$ look-up tables have been identified. The two bounded input signals u_{VTG} and u_{EGR} are normalized to the range of $[0, 100]$.

Some characteristics of the functions will be very helpful for the observability analysis in Section III: The flow function $\Phi(p_{\mathrm{IM}}, n_{\mathrm{TC}})$ is strictly monotonic increasing in n_{TC} for $p_{\mathrm{IM}} > 0$ and the mass flow W_{T} is strictly monotonic increasing in p_{OM} for all $u_{\mathrm{VTG}} \in [0, 100]$. For more details on the structure of Φ and Ψ see [6], [8], respectively.

VARIABLES

p_{IM}	compressor downstream pressure (intake manifold)
p_{OM}	turbine upstream pressure (exhaust manifold)
n_{TC}	rotational speed of turbine
T_{UT}	turbine upstream temperature
T_{IM}	intake manifold temperature
T_{OM}	cylinder exit temperature
W_{C}	air mass flow through compressor
W_{T}	air mass flow through turbine
W_{EGR}	air mass flow through exhaust gas recirculation
$W_{\mathrm{cyl}}^{\mathrm{in}}$	mass flow into cylinders
$W_{\mathrm{cyl}}^{\mathrm{out}}$	mass flow out of cylinders
u_{VTG}	variable turbine geometry input
u_{EGR}	exhaust gas recirculation input

LOOK-UP TABLES

$A_{\mathrm{VTG}}(u_{\mathrm{VTG}})$	variable turbine geometry effective area
$A_{\mathrm{EGR}}(u_{\mathrm{EGR}})$	exhaust gas recirculation effective area
$\Phi(p_{\mathrm{IM}}, n_{\mathrm{TC}})$	compressor flow factor
$\Psi(p_{\mathrm{OM}}, u_{\mathrm{VTG}})$	turbine power factor
$\eta_{\mathrm{T}}(u_{\mathrm{VTG}})$	turbine efficiency degree
$\eta_{\mathrm{C}}(p_{\mathrm{IM}}, W_{\mathrm{C}})$	compressor efficiency degree

CONSTANT PARAMETERS

R	ideal gas constant ($R = 8.314$)
κ	isentropic exponent (air: $\kappa = 1.4$)
p_{amb}	ambient pressure
V_{IM}	intake manifold volume
V_{EM}	exhaust manifold volume
η_{m}	mechanical efficiency degree of turbo charger
J	inertial moment of the turbo charger
τ_{VTG}	VTG-actuator time constant
τ_{EGR}	EGR-actuator time constant

Beside the highly nonlinear dynamics of the presented system model and the look-up tables, there is another difficulty for the observability analysis. The functions Φ and Ψ as well as the EGR mass-flow W_{EGR} contain square-root expressions which are not Lipschitz-continuous. In particular for the practically relevant point $p_{\mathrm{IM}} = p_{\mathrm{OM}}$, no Lipschitz constant exists.

In this paper we shall consider two different models of the turbocharger: system Σ^1 containing the full dynamics (1)-(6), and the reduced-order system Σ^2. Each system is of the form:

$$\Sigma^i : \begin{cases} \dot{x}^i = F^i(x^i, u) = f^i(x^i) + g^i(x^i, u), \\ y^i = h^i(x^i) = p_{\mathrm{IM}}, \end{cases} \tag{7}$$

with $F^i : \mathbb{R}^{n^i} \times \mathbb{R}^2 \to \mathbb{R}^{n^i}$ and $h^i : \mathbb{R}^{n^i} \to \mathbb{R}$ sufficiently often differentiable in x^i and $u = (u_{\mathrm{VTG}}^{\mathrm{ref}}, u_{\mathrm{EGR}}^{\mathrm{ref}})^\top$. Note that the proposed system Σ^i can be separated into a homogeneous part f^i and an input dependent part g^i.

For Σ^1, F^1 is given by the right-hand sides of (1) - (6) with $n^1 = 6$ and state-vector

$$x^1 = \left(p_{\mathrm{IM}},\, n_{\mathrm{TC}},\, p_{\mathrm{OM}},\, T_{\mathrm{OM}},\, u_{\mathrm{VTG}},\, u_{\mathrm{EGR}}\right)^\top.$$

For the analytic observability analysis we consider the reduced-order model Σ^2. Here the actuator dynamics are neglected. Thus we use the inputs $u_{\mathrm{VTG}} = u_{\mathrm{VTG}}^{\mathrm{ref}}$ and

$u_{\mathrm{EGR}} = u_{\mathrm{EGR}}^{\mathrm{ref}}$ for F^2. Also, we neglect the dynamics of the turbine up-stream temperature T_{UT}. This results for Σ^2 in $n^2 = 3$ with F^2 given by the right-hand sides of (1) - (3) with

$$x^2 = \left(p_{\mathrm{IM}},\, n_{\mathrm{TC}},\, p_{\mathrm{OM}}\right)^\top .$$

III. OBSERVABILITY

In general, the observability analysis of a nonlinear system is a rather difficult task and cannot be performed as easily as in the linear case. There exist various types of observability concepts that can be applied. In this contribution we consider the *weak observability (WO)* and the *uniform observability (UO)*, which are significant for the usage of the observer approaches that we shall apply.

Consider the general single-output system class Σ^i from (7). The related observability map $\mathfrak{o}_k : \mathbb{R}^{n^i} \to \mathbb{R}^k$ with respect to F^i is defined as in [9]:

$$\begin{aligned}\mathfrak{o}_k(x^i) &:= \begin{pmatrix} y^i & \dot{y}^i & \ddot{y}^i & \cdots & y^{i(k)} \end{pmatrix}^\top \\ &= \begin{pmatrix} h^i(x^i) & \mathcal{L}_{F^i} h^i(x^i) & \cdots & \mathcal{L}_{F^i}^k h^i(x^i) \end{pmatrix}^\top ,\end{aligned}$$

where $\mathcal{L}_{F^i}^k$ describes the k-th Lie derivative with respect to $F^i(\cdot, u)$. The aim of characterizing the observability of a system is to determine the solvability of the resulting system of nonlinear equations for the state x^i. In view of the *Implicit Function Theorem*, the Jacobian of this map can be used to show *local solvability* of these equations. The Jacobian of the observability map is called *observability matrix* $\mathcal{O}_k(x^i) \in \mathbb{R}^{k \times n^i}$ and can be calculated in the following way:

$$\mathcal{O}_k(x) := \frac{\partial \mathfrak{o}_k}{\partial x^i}(x^i) = \begin{pmatrix} \frac{\partial}{\partial x^i} h^i(x^i) \\ \frac{\partial}{\partial x^i} \mathcal{L}_{F^i} h^i(x^i) \\ \frac{\partial}{\partial x^i} \mathcal{L}_{F^i}^2 h^i(x^i) \\ \vdots \\ \frac{\partial}{\partial x^i} \mathcal{L}_{F^i}^k h^i(x^i) \end{pmatrix} ,$$

where $\frac{\partial}{\partial x^i}(\cdot)$ denotes the gradient (row vector).

To show WO the following observability rank condition from [4] is applied:

Theorem 1: The system Σ^i from (7) is *locally weakly observable (LWO) at* $x_0 \in \mathbb{R}^{n^i}$ if $\exists k \in \mathbb{N}$ such that for all constant $u \in \mathbb{R}^2$: $\mathcal{O}_k(x_0) \in \mathbb{R}^{k \times n^i}$ and

$$rk\big(\mathcal{O}_k(x_0)\big) = n^i. \tag{8}$$

If this holds for all $x_0 \in \mathbb{R}^{n^i}$ the system Σ^i is *globally weakly observable*.

Calculating the observability matrix for our systems Σ^i analytically, yields rather lengthy expressions that are not easy to evaluate. Therefore we choose to evaluate the observability rank condition (8) numerically for $k = n^i - 1$. Fig. 2 shows the results in the p_{IM}-n_{TC}-plane. Points for which the observability condition fails are marked red. Both systems Σ^i, $i \in \{1, 2\}$ show very similar results.

Fig. 2 also contains measurement data of dynamic trajectories from an engine test-rig (blue). This shows how close the actual system can get to an unobservable condition.

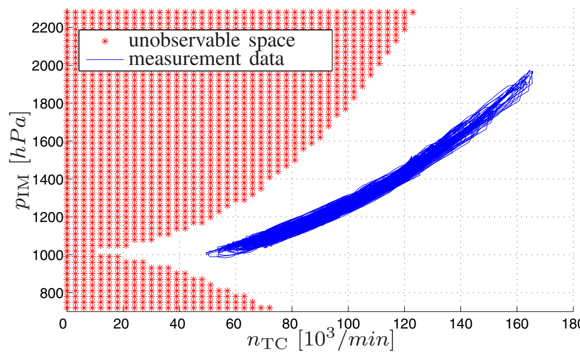

Fig. 2. $p_{\mathrm{IM}} - n_{\mathrm{TC}}$ projection of the state space. red: unobservable subspace, i.e. (8) fails, blue: dynamic measurement data from test rig.

Interestingly, the lack of observability is directly related to the compressor mass flow W_{C} becoming zero. Such situation is not expected to occur in practice. However, the mass flow W_{C} highly depends on the identification of the flow function Φ which is non-Lipschitz at the boundary of the illustrated unobservable space. Therefore, large initial estimation errors may cause the observer itself to enter the unobservable region which may imperil the convergence of the estimation error.

As already discussed in Section II the EGR pressure ratio is very important. Due to the non-Lipschitz point in W_{EGR} at $p_{\mathrm{IM}} = p_{\mathrm{OM}}$ the condition $p_{\mathrm{OM}} > p_{\mathrm{IM}}$ is necessary for the existence of $\mathcal{O}_{n^i-1}(x_0)$. Further, the pressure p_{OM} has to be larger than the ambient air pressure p_{amb} to insure the differentiability of the turbine power factor Ψ. Summing up, both systems Σ^i, $i \in \{1, 2\}$ are LWO on the set

$$\Omega^i = \Big\{ x^i \,\Big|\, p_{\mathrm{OM}} > p_{\mathrm{IM}} > 0,\, p_{\mathrm{OM}} > p_{\mathrm{amb}},\, \Phi(p_{\mathrm{IM}}, n_{\mathrm{TC}}) > 0 \Big\}.$$

After checking the geometrical condition for WO numerically, we shall now consider the stronger concept of *uniform observability (UO)*, analytically. Following [9], the system is UO if the systems of equations given by the observability map $\mathfrak{o}_k(x^i)$ has a unique solution for the state for any given input. In particular it is required that there does not exist a so-called singular input which renders the state reconstruction infeasible. Such input may cause for undistinguishable conditions and thus an unique solution to the observation problem may not exist.

For simplicity, consider system Σ^2 with EGR turned off, i.e. $A_{\mathrm{EGR}} = 0$. Then the components of the observability map $\mathfrak{o}_k(x^i)$ have the following form:

$$y^2 = p_{\mathrm{IM}}, \tag{9}$$

$$\dot{y}^2 = -k_1\, p_{\mathrm{IM}} + k_3\, \Phi(p_{\mathrm{IM}}, n_{\mathrm{TC}})\, n_{\mathrm{TC}}, \tag{10}$$

$$\begin{aligned}\ddot{y}^2 = &\left(-k_1 + k_3\, n_{\mathrm{TC}} \frac{\partial \Phi}{\partial p_{\mathrm{IM}}}\right) \dot{h} + k_3 \left(\frac{\partial \Phi}{\partial n_{\mathrm{TC}}} n_{\mathrm{TC}} + \Phi\right) \times \\ &\times \left[\frac{k_{\mathrm{T}}}{n_{\mathrm{TC}}} \Psi \left(1 - \left(\frac{p_{\mathrm{amb}}}{p_{\mathrm{OM}}}\right)^{\frac{\kappa-1}{\kappa}}\right) p_{\mathrm{OM}}\, A_{\mathrm{VTG}}\, \eta_{\mathrm{T}} - \right. \\ &\left. - k_{\mathrm{C}} \frac{\Phi(p_{\mathrm{IM}}, n_{\mathrm{TC}})}{\eta_{\mathrm{C}}} \left(\left(\frac{p_{\mathrm{IM}}}{p_{\mathrm{amb}}}\right)^{\frac{\kappa-1}{\kappa}} - 1\right)\right]. \end{aligned} \tag{11}$$

While a readily applicable condition for the non-existence of a singular input has yet to be found, the solvability of (9)-(11) for $x^2 \in \Omega^2$ and any possible input A_{VTG} has to be analyzed directly. For this analysis y^2, $\dot{y}^2$ and $\ddot{y}^2$ are assumed known. We shall use the triangular structure and the monotonicity properties.

The first equation (9) yields the state p_{IM}. Then the second equation (10) only depends on n_{TC}. Due to the strict monotonicity of $\Phi(p_{\mathrm{IM}}, n_{\mathrm{TC}})n_{\mathrm{TC}}$ in n_{TC}, the unique solution for the state n_{TC} can be obtained. In (11) only the turbine power relation is dependent on the remaining state p_{OM}. For $p_{\mathrm{OM}} > p_{\mathrm{amb}}$ and with the input satisfying $A_{\mathrm{VTG}}(u_{\mathrm{VTG}})\,\eta_{\mathrm{T}} \neq 0$ for all u_{VTG}, this term is strictly monotonic in p_{OM} and thus we find a unique solution for p_{OM}. This proves the UO of the system with EGR turned off.

For the case of non-zero EGR there are a lot of couplings between the state dynamics and the triangular structure is lost. Thus, UO is hard to show analytically. However, during extensive evaluations using simulation data from the full-order model with various driving conditions no indistinguishable states could be detected.

IV. HIGH-GAIN OBSERVER

The main idea of the High-Gain (HG) observer is to find a linear output injection which is able to dominate the nonlinear terms and stabilizes the estimation error dynamics. It is relatively easy to design and has useful robustness properties. Additionally it allows to include the given system dynamics into the observer structure. In particular the non-Lipschitz terms may be treated as disturbances such that no evaluation of their derivative is needed. This makes the HG observer applicable to the turbocharger system Σ^i.

Following [10] and [11] the High-Gain observer is designed for systems of the form

$$\begin{aligned} \dot{z} &= A_0 z + \psi(z,u) + e_{n^i} v(t) \\ y &= C_0 z + w(t) = z_1 + w(t) \end{aligned} \tag{12}$$

where $A_0 \in \mathbb{R}^{n^i \times n^i}$ with elements $a_{jk} = \delta_{j+1,k}$, $\delta_{j,k}$ the Kronecker delta and ψ is a nonlinearity supposed to be Lipschitz continuous in z with $\psi_j = \psi_j(z_1, \ldots, z_j, u)$. In order to investigate robustness of the estimation, the bounded disturbances v and w with $v(t) \leq v_\infty$ and $w(t) \leq w_\infty$, $\forall t \geq 0$ are considered.

The transformation of the system Σ^i into the form (12) is obtained from the observability map $z := \mathfrak{o}_{n^i-1}(x^i)$ with respect to the homogeneous system part f^i as in [11] ("drift system"). Of course this requires the regularity of the corresponding observability matrix $\mathcal{O}_{n^i-1}(x^i)$ for every x^i contained in the considered state space.

It has been shown in [5] that the existence of such a diffeomorphic change of coordinates is equivalent to *uniform observability* of the system. This means there are no distinct input signals for which the resulting state trajectories cannot be distinguished as described in [12]. It refers to the importance of inputs for the observability of nonlinear systems and forms a key requirement for the convergence of the High-Gain observer.

For system (12) the following observer approach is considered

$$\begin{aligned} \dot{\hat{z}} &= A_0 \hat{z} + \psi(\hat{z},u) + \Lambda H_0 \left(y - C_0 \hat{z}\right) \\ \hat{x}^i &= \mathfrak{o}^{-1}_{n^i-1}(\hat{z}) \end{aligned} \tag{13}$$

with $H_0 \in \mathbb{R}^{n^i}$, $\Lambda \in \mathbb{R}^{n^i \times n^i}$ and $\Lambda = \mathrm{diag}(L, L^2, \ldots, L^{n^i})$, $L > 0$. To ensure stability of the observer error dynamics, the remaining parameters are selected as stated in [10] with $\gamma > 0$ being the Lipschitz constant of $\psi(\cdot, u)$:

- H_0 such that $A_0 - H_0 C_0$ Hurwitz,
- L sufficiently large, i.e. $L > L_0 = 2\lambda_0\gamma$.

While the tuning parameter L can be chosen arbitrarily large, it is effected by the so-called *peaking phenomenon*. This can be characterized by the following boundary of the estimation error $e := z - \hat{z}$:

$$\|e(t)\| \leq c_P \exp\left[-\frac{1}{2}\gamma(L - L_0)t\right] L^{n^i-1}\|e_0\| + \\ + \frac{2c_P^2\|H_0\|L^n}{\gamma(L-L_0)} w_\infty + \frac{2c_P^2}{\gamma(L-L_0)} v_\infty \;, \forall t \geq 0$$

with c_P as well as λ_0 being constants dependent on the choice of H_0. For increasing L the following effects occur:

- the observation error converges faster to zero,
- the initial observation error becomes larger, i.e. may result in large estimation errors in the beginning,
- the measurement noise is amplified,
- the impact of process disturbances is weakened.

This trade-off is the essential for the design procedure and has to be kept in mind. Another problem for the observer realization is the inversion of the observability map $\mathfrak{o}_{n^i-1}$ in general and in particular for the turbocharger system. To avoid this difficult task the following alternative observer realization, equivalent to (13), is applied:

$$\dot{\hat{x}}^i = f^i(\hat{x}^i) + g^i(\hat{x}^i, u) + \mathcal{O}^{-1}_{n^i-1}(\hat{x}^i)\Lambda H_0\big(y^i - h^i(\hat{x}^i)\big)\,. \tag{14}$$

Here only the much simpler matrix inversion of the observability matrix $\mathcal{O}_{n^i-1}(\hat{x}^i)$ is required. The existence of the inverse is already guaranteed due to the regularity of the matrix. In practice, this is ensured by checking the rank condition (8) for the homogeneous system part. So $\dot{x}^i = f^i(x^i),\ y^i = h^i(x^i)$ from (7) has to be locally weakly observable.

The observability analysis in Section III shows that the system in realistic driving conditions can be close to an unobservable state, c.f. Fig. 2. Thus, the High-Gain observer may enter the unobservable region during the initial transient caused by the peaking phenomenon. Therefore, the observer gain L has to be chosen with care. Note also that the gain L is risen to the power of the system order n_i. Thus the reduced-order system Σ^2 is less prone to violate the observability region.

As we are mainly interested in estimating the turbocharger rotation speed n_{TC}, we can reduce the system order even further by applying the observer only to the states p_{IM}

and n_{TC}. The third state p_{OM} is obtained by pure simulation. The evaluation shows that the robustness property of the HG observer can compensate the resulting simulation errors.

Another difficulty are the non-Lipschitz points that occur due to the EGR term. Especially here the High-Gain observer has its advantage. As the EGR term is part of the inhomogeneity $g^i(x^i, u)$ it is not used to calculate the observability matrix $\mathcal{O}_{n^i-1}(\hat{x}^i)$ and thus no differentiation is needed. Instead it only appears in $\psi(z, u)$ in connection with the transformed system (13).

V. EXTENDED KALMAN FILTER

The standard approach for most industry applications, the Kalman-Bucy filter, shall also be discussed here. For nonlinear systems the respective Extended Kalman filter usually is obtained by analytic linearization, see e.g. [13]. However, the analyzed system model of the turbo-charged Diesel engine contains square root terms whose derivatives do not exist at some points. For this reason this technique can not be directly applied to our system class. In the following, we present a pragmatic approach to deal with this problem by numerical differentiation such that the critical derivatives are replaced with some rough approximation.

Consider the general nonlinear system Σ^i from (7). The suboptimal Extended Kalman filter (EKF), as presented in [9], has the following structure:

$$\dot{\hat{x}}^i = F^i(\hat{x}^i, u) + K\left(y^i - h^i(\hat{x}^i)\right) \ , \ \hat{x}^i(0) = \hat{x}^i_0, \qquad (15)$$

where $K = PC^\top R^{-1}$ and with the dynamics of covariance $P \in \mathbb{R}^{n^i \times n^i}$:

$$\dot{P} = AP + PA^\top - PC^\top R^{-1} CP + Q + \delta P \ , \ P(0) = P_0 \, ,$$

where $Q \in \mathbb{R}^{n^i \times n^i}$, $Q = Q^\top \geq 0$ denotes the process noise covariance matrix and the scalar $R > 0$ denotes the measurement noise variance. Typically these matrices together with the parameter $\delta \geq 0$ are chosen for tuning the EKF.

The linearization terms can not be obtained by analytically calculating the Jacobian because of the lacking Lipschitz continuity at some points. Therefore the matrices

$$A = \frac{\partial F^i}{\partial x^i}(\hat{x}^i, u) \, , \qquad C = \frac{\partial h^i}{\partial x^i}(\hat{x}^i)$$

are obtained with help of a finite difference approximation

$$\frac{\partial F^i_k}{\partial x^i_j}(\hat{x}^i, u) \approx \frac{F^i_k\left([\hat{x}^i_1, ..., \hat{x}^i_j + h_j, ..., \hat{x}^i_{n_i}], u\right) - F^i_k(\hat{x}^i, u)}{h_j},$$

where x^i_j indicates the j-th element of x^i and $k, j \in \{1, \ldots, n^i\}$. For adjusting the numerical requirements of each state due to the different scalings, different step-sizes $h = (h_1, \ldots, h_{n^i})^\top$ are used. The difference quotient is relatively easy to compute and just requires evaluations of the function F^i. Here, a balance between accuracy of the linearization and sufficiently rough approximation for stability has to be made.

Another more precise technique for the indirect linearization is the complex step differentiation introduced by [14]. There, complex arithmetic is used to obtain a high-order approximation of the respective derivative. Unfortunately we can not apply this technique here since look-up tables have been used for the parameterized turbocharger model.

Convergence of the EKF is difficult to show due to the linearization approximation. If the system Σ^i is uniformly observable and the Hessian (second derivative) of the nonlinearity is bounded then convergence of the EKF can be shown, [15]. However, due to the non-differentiable points the latter can not be be guaranteed globally in our case.

Due to the simplicity of the algorithm we can base the EKF upon the full model Σ^1. To handle the non-Lipschitz points that may occur occasionally, a sufficiently large step size h_j has to be selected. Indeed, this decreases the numerical precision but makes the algorithm less sensitive against non-differentiability. Our evaluation shows that the resulting inaccuracy can be handled sufficiently well by the robustness of the EKF against Gaussian type perturbations.

VI. EVALUATION

A diesel engine test-rig is used for generating real-time measurements as a reference and data basis for the evaluation of the respective estimators. The data processing and testing of the observation methods has been performed with MATLAB Simulink. The main objective of this study is the estimation of the turbocharger rotation speed n_{TC} only using measurements of the pressure p_{IM}. Fig. 3 shows the absolute estimation error $\Delta n_{\mathrm{TC}} = n_{\mathrm{TC}} - \hat{n}_{\mathrm{TC}}$ of the full-order model Σ^1 (1)-(6), the High-Gain (HG) observer (14) based on reduced-order model Σ^2 and the Extended Kalman filter (EKF) from (15) based on full-order model Σ^1.

Both observers show significantly better performance compared to the estimation obtained by the full-order model Σ^1. The EKF shows better results than the HG approach. This is mainly due to the more comprehensive model used for the EKF. Comparisons of an EKF based on the reduced-order system Σ^2 show very similar results to the HG observer. Thus, the model used is of larger significance for the estimation error than the observer methods considered. Further investigations with different levels of dynamic simplifications show that indeed the actuator dynamics (5) and (6) have the

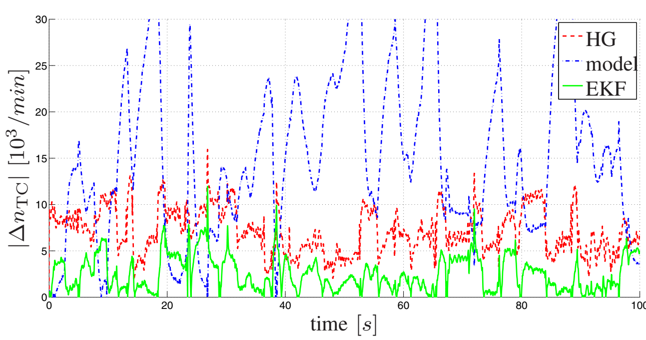

Fig. 3. Estimation error $|\Delta n_{\mathrm{TC}}| = |n_{\mathrm{TC}} - \hat{n}_{\mathrm{TC}}|$ using test-rig data: simulation (blue) Σ^1, HG observer (red) from (14) based on reduced-order model Σ^2, and EKF (green) from (15) based on full-order model Σ^1.

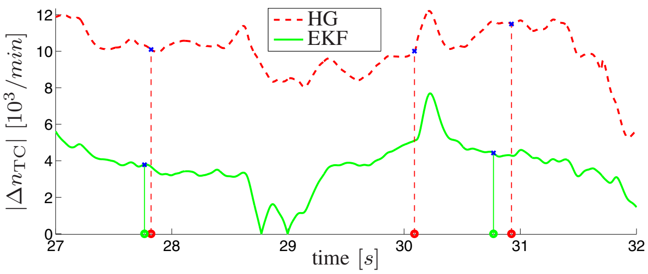

Fig. 4. Detail of Fig. 3 with points where $\hat{p}_{\mathrm{IM}}=\hat{p}_{\mathrm{OM}}$ (non-Lipschitz).

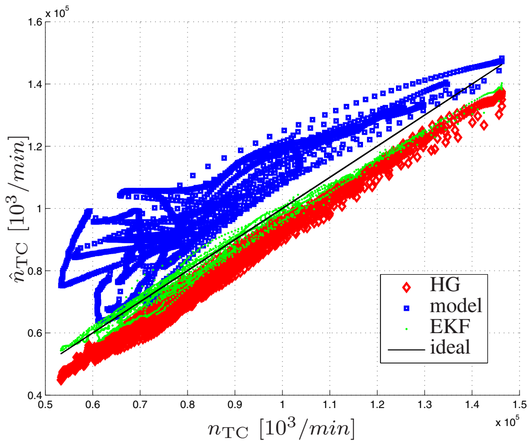

Fig. 5. Estimated state $\hat{n}_{\mathrm{TC}}$ over true value n_{TC}: simulation (blue) Σ^1, HG observer (red) from (14) based on reduced-order model Σ^2, and EKF (green) from (15) based on full-order model Σ^1.

most significant impact, while adding just the temperature dynamics (4) does not alter the performance significantly. This confirms the findings in [6].

Fig. 4 shows a detail of Fig. 3. The points where the respective observer reaches the non-Lipschitz condition $\hat{p}_{\mathrm{IM}} = \hat{p}_{\mathrm{OM}}$ are marked. The evaluation shows, that the non-Lipschitz continuous points of the system do not impair stability of the estimation. The reason for this may be that the Lipschitz condition is only sufficient for state estimation convergence but not necessary. In view of this, the two pragmatic approaches manage to deal with this problem by not including direct derivatives of the respective terms. Other methods using direct differentiation, like the conventional EKF, would have singularities at these points.

Fig. 5 shows the relation between the observed state $\hat{n}_{\mathrm{TC}}$ and actual state n_{TC}. The HG observer shows an offset which is due to the neglected dynamics. The full-order model shows a poor performance for low speeds n_{TC}. For large speeds, both observers are slightly worse than the model.

VII. CONCLUSION

The analysis of the observability of a turbocharged diesel engine air path system with focus on estimating the turbocharger speed has been performed in three steps. First, parameterized models containing nonlinear terms, look-up tables and non-Lipschitz continuous points are presented. Secondly, weak and uniform observability are analyzed. It turns out that practically relevant state conditions are close to the unobservable space. Finally, a High-Gain observer and EKF are designed to estimate the system states. Both are suitable for the nonlinear system structure and avoid direct differentiation of the non-Lipschitz terms.

The evaluations using experimental test-data show that the two observers are able to reconstruct the desired state much better than the system model. There is no evidence of any malicious effect by the non-Lipschitz points as would be expected using estimation methods relying on direct differentiation of the dynamics.

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REFERENCES

[1] B. C. Glenn, D. Upadhyay, V. I. Utkin, G. N. Washington, and M. B. Hopka, “Observer design of critical states for air path flow regulation in a variable geometry turbocharger exhaust gas recirculation diesel engine,” *Int. J. of Engine Research*, vol. 12, no. 6, pp. 501–512, 2011.

[2] R. Salehi, M. Shahbakhti, A. Alasty, and G. Vossoughi, “Nonlinear observer design for turbocharger in a si engine,” in *American Control Conference (ACC)*, Washington, DC, June 2013, pp. 5231–5236.

[3] Q. Qu, H. Wang, and Y. Tian, “Nonlinear observer based sliding mode control for a turbocharged diesel engine air-path equipped with egr and vgt,” in *Chinese Automation Congress*, Wuhan, 2015, pp. 121 – 126.

[4] R. Hermann and A. J. Krener, “Nonlinear controllability and observability,” *IEEE Transactions on Automatic Control*, vol. 22, no. 5, pp. 728–740, 1977.

[5] J. P. Gauthier and G. Bornard, “Observability for any u(t) of a class of nonlinear systems,” *IEEE Transactions on Automatic Control*, vol. 26, no. 4, pp. 922–926, 1981.

[6] J. Wahlström and L. Eriksson, “Modelling diesel engines with a variable-geometry turbocharger and exhaust gas recirculation by optimization of model parameters for capturing non-linear system dynamics,” *J. of Autom. Engineering*, vol. 225, no. 7, pp. 960–986, 2011.

[7] L. Guzzella and C. H. Onder, *Introduction to Modeling and Control of Internal Combustion Engine Systems*, 2nd ed. Berlin Heidelberg: Springer-Verlag, 2010.

[8] M. Schollmeyer, “Beitrag zur modellbasierten Ladedruckregelung für Pkw-Dieselmotoren,” PhD Thesis, Leibniz Universität Hannover, Aachen, 2010.

[9] G. Besançon, “An overview on observer tools for nonlinear systems,” in *Nonlinear observers and applications*, ser. Lecture notes in control and information sciences, G. Besançon, Ed. Berlin: Springer, 2007, vol. 363, ch. 1, pp. 1–33.

[10] H. Hammouri, “Uniform observability and observer synthesis,” in *Nonlinear observers and applications*, ser. Lecture notes in control and information sciences, G. Besançon, Ed. Berlin: Springer, 2007, vol. 363, pp. 35–69.

[11] J. P. Gauthier, H. Hammouri, and S. Othman, “A simple observer for nonlinear systems applications to bioreactors,” *Automatic Control, IEEE Transactions on*, vol. 37, no. 6, pp. 875–880, 1992.

[12] F. C.-C. G. Bornard and G. Gilles, “Observability and observers,” in *Nonlinear Systems - Modeling and Estimation*, A. J. Fossard, Ed., vol. 1. London: Chapman & Hall, 1995, pp. 173–216.

[13] A. Gelb, *Applied optimal estimation*. Cambrdg, Mass.: MIT Press, 1992.

[14] W. Squire and G. Trapp, “Using complex variables to estimate derivatives of real functions,” *SIAM Review*, vol. 40, no. 1, pp. 110–112, 1998.

[15] A. J. Krener, “The convergence of the extended kalman filter,” in *Directions in Mathematical Systems Theory and Optimization*, ser. LNCIS, A. Rantzer and C. Byrnes, Eds. Springer-Verlag Berlin Heidelberg, 2003, no. 286, ch. 12, pp. 173–182.