A boost unity power factor pre-compensator

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***Abstract*— In this contribution, a passivity-based output feedback controller is developed for the unity-power-factor pre-compensation of a boost-type AC/DC power converter. In such applications, aside from load voltage tracking to a pre-specified initial and final constant level, a vital additional control objective consists in keeping the input power factor close to unity. By means of this controller, the desired input current of the converter has been designed for being in phase with the supply voltage. The control law is synthesized using a linear time-varying combination of the converter input current and voltage variables. The experimental results underscore that the controlled tracking performance of the output voltage is satisfactory, even under unity power factor requirements.**

I. Introduction

The single-phase boost rectifier has become the most popular topology for power factor correction (PFC) in general purpose power supplies. Today there is interest in developing high-frequency electronics ballast to replace the classical electro-magnetic ballast in fluorescent lamps. These electronics ballasts require an AC/DC converter. To satisfy the harmonic current injection from electronic equipment and to maintain high power quality, a high-power factor rectifier may be used [1]. In this context, some new single-phase rectifiers (AC/DC) topologies came up for feeding DC loads in such a way that the AC current supply line shows a high power factor and low current harmonics [2], [3]. AC-to-DC converters are circuits that alleviate the main drawbacks found in DC/DC converters, namely, the low input-power factor and the considerable pollution introduced in the line source [4].

The performance of power converters may be enhanced remarkably via nonlinear control techniques. As opposed to conventional linear control based on the linearization about an equilibrium point, nonlinear techniques allow to operate the system within the full range of model validity, just resorting to a single control law. Under mild restrictions, such-like controllers may also assure a smooth transition between equilibria while showing neither overshoot nor undershoot.

Here, based on the average system model of Boost-type AC/DC-converter, we develop a unity power factor controller that stabilizes the output voltage to a constant reference value. To this end, a so-called flatness property of the system is exploited so as to generate valid reference trajectories for the nominal control (see [5]). The feedback part of the control is devised resorting to a recently introduced control policy, named exact tracking error dynamics passive output feedback control [6], [7], [8], where tuning just requires to specify one single parameter. Regarding applications, the presented control scheme may be used, for instance, for the gradual regulation of DC lamps (dimmers) and also as a smooth starter for DC machines (variable speed).

The outline of the paper is as follows: The average system model and the objectives for the control of a boost type unity power factor rectifier are presented in Section II. A procedure for generating valid reference trajectories for the nominal control is developed in Section III. The derivation of the feedback control law of the exact tracking error dynamics passive output feedback controller is given in Section IV. Experimental results from a real laboratory setup are provided in Section V. Conclusions and perspectives may be found in Section VI.

II. A boost unity power factor pre-compensator

Consider the *boost type unity power factor rectifier*, or mono-phasic rectifier, shown in Fig. 1.

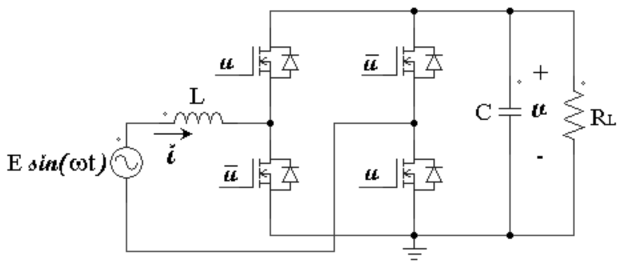

Fig. 1. Power factor pre-compensator boost circuit

The underlying dynamical system is described by the following set of differential equations:

$$L\frac{di}{dt} = -u\,v + E\sin(\omega t) \tag{1}$$

$$C\frac{dv}{dt} = u\,i - \frac{1}{R_{\rm L}}v \tag{2}$$

where i is the inductor current, v is the output capacitor voltage, and u is the switch position function that takes values in the discrete set $\{-1, 1\}$. The line resistance was assumed negligible. Assuming a high-frequency switching policy, we consider the *state average* system equations by simply replacing the actual states of the system description by average states, while letting the control input take continuous values in $[-1, 1]$. In other words, we assume the *average model*

$$L\frac{di}{dt} = -u_{\rm av}\, v + E\sin(\omega t) \tag{3}$$

$$C\frac{dv}{dt} = u_{\rm av}\, i - \frac{1}{R_{\rm L}} v \tag{4}$$

where the continuous control signal $u_{\rm av}$ takes values in $[-1, 1]$.

The control objectives are twofold: First, it is desired to have the average inductor current variable track a sinusoidal signal of constant amplitude A, yet to be determined, and of the same angular frequently ω as the input source. This objective guarantees a unity power factor. It is also desired that the DC component of the average voltage v stabilizes to a constant reference value, denoted by $V_{\rm d}$.

Let us consider the total stored average energy of the system (3)-(4), given by

$$H = \frac{1}{2}L\, i^2 + \frac{1}{2}C\, v^2\,. \tag{5}$$

The total power is given by the time derivative of H, i.e.

$$\frac{dH}{dt} = i\, E\sin(\omega t) - \frac{1}{R_{\rm L}}v^2 \tag{6}$$

where the first summand represents the input power and the second term corresponds to the delivered power at the load. The steady state value of the DC component of the total power should balance to zero, since the system is lossless.

Therefore, we have the following steady state power balance condition

$$\langle \bar{\imath}\, E\sin(\omega t)\rangle_{\rm dc} = \left\langle \frac{1}{R_{\rm L}}\bar{v}^2 \right\rangle_{\rm dc} \tag{7}$$

in which "overline" denotes the steady state value of the involved variable.

Using the desired DC-component value $\langle \bar{v}\rangle_{\rm dc} = V_{\rm d}$ as the steady state value the relationship

$$\left\langle AE\sin^2(\omega t)\right\rangle_{\rm dc} = \frac{V_{\rm d}^2}{R_{\rm L}}$$

is obtained from where it is immediate to see

$$A = \frac{2V_{\rm d}^2}{R_{\rm L}E}\,. \tag{8}$$

The fact that the inductor current amplitude A and the desired DC component of the output voltage satisfy the above relation is often called the *solvability condition*. When considering inductor resistances, this condition reveals a natural limit on the reachable output voltages.

III. Trajectory generation

The generation of the desired references $i^\star$, $v^\star$, $u_{\rm av}^\star$ that are needed for the feedback controller is based on the accordance with the control objectives. The problem is by no means a trivial one unless one resorts to the flatness property of the average model [5]. In the nominal case, the average model of the boost unity power factor pre-compensator reads

$$L\frac{di^\star}{dt} = -u_{\rm av}^\star v^\star + E\sin(\omega t) \tag{9}$$

$$C\frac{dv^\star}{dt} = u_{\rm av}^\star i^\star - \frac{1}{R_{\rm L}}v^\star \tag{10}$$

In [5] it is shown that a so-called flat output is given by the desired average total stored energy:

$$F^\star = \frac{1}{2}L\left(i^\star\right)^2 + \frac{1}{2}C\left(v^\star\right)^2\,. \tag{11}$$

The time derivative of F^*, which is the desired total average power, is then given by

$$\dot{F}^\star = i^\star E\sin(\omega t) - \frac{1}{R_{\rm L}}\left(v^\star\right)^2\,. \tag{12}$$

Thus for a unity power factor, the nominal value of $i^\star(t)$ needs to be chosen as

$$i^\star(t) = A\sin(\omega t) \tag{13}$$

and solving equation (11) for $(v^\star)^2$ leads to the following (stable) differential equation for $F^\star$:

$$\dot{F}^\star = -\frac{2}{R_{\rm L}C}F^\star + A\left(E + \frac{LA}{R_{\rm L}C}\right)\sin^2(\omega t)\,. \tag{14}$$

In terms of the desired steady state constant average output voltage $\langle \bar{v}\rangle_{\rm dc} = V_{\rm d}$ the differential equation satisfied by the flat output (average total stored energy) results from (8), hence

$$\dot{F}^\star = -\frac{2}{R_{\rm L}C}F^\star + \frac{2V_{\rm d}^2}{R_{\rm L}E}\left(E + \frac{2V_{\rm d}^2 L}{R_{\rm L}^2 EC}\right)\sin^2(\omega t)\,. \tag{15}$$

A few steps of simple calculations show that the DC component of the steady state solution of the above differential equation (15) reads

$$\left\langle \bar{F}\right\rangle_{\rm dc} = \frac{V_{\rm d}^2}{2}\left(C + \frac{2V_{\rm d}^2 L}{R_{\rm L}^2 E^2}\right) \tag{16}$$

which precisely coincides with the value obtained from the reference flat output definition (11)

$$\begin{aligned}\left\langle \bar{F}\right\rangle_{\rm dc} &= \frac{1}{2}\left(L\left\langle \bar{\imath}^2\right\rangle_{\rm dc} + C\left\langle \bar{v}^2\right\rangle_{\rm dc}\right)\\ &= \frac{1}{2}\left(L\left\langle A^2\sin^2(\omega t)\right\rangle_{\rm dc} + C\, V_{\rm d}^2\right)\\ &= \frac{V_{\rm d}^2}{2}\left(C + \frac{2V_{\rm d}^2 L}{R_{\rm L}^2 E^2}\right)\,.\end{aligned} \tag{17}$$

The trajectory planning aspects of the problem, aimed at producing the required nominal state and control input trajectories, is carried out as follows:

First of all, we specify a desired steady state average output voltage $V_{\rm d}$. Thanks to equation (8), this quantity allows to determine the average line current $i = A\sin(\omega t)$. With the value of A and of $V_{\rm d}$ we may compute the average value of the nominal steady state flat output, $\left\langle \bar{F}\right\rangle_{\rm dc}$, as expressed in equation (16).

Given an equilibrium value of the average capacitor voltage, denoted by $\bar{V}_{\text{d,init}}$ and valid up to time t_{init}, and a desired final equilibrium value of this voltage, $\bar{V}_{\text{d,final}}$ valid only after t_{final}, with $t_{\text{final}} > t_{\text{init}}$, we may then specify a nominal trajectory of time for the flat output. Such a trajectory is specified to smoothly interpolate between the corresponding values

$$\bar{F}_{\text{init}} = \left(\langle \bar{F} \rangle_{\text{dc}}\right)\Big|_{V_{\text{d}} = \bar{V}_{\text{d,init}}} \tag{18}$$

and

$$\bar{F}_{\text{final}} = \left(\langle \bar{F} \rangle_{\text{dc}}\right)\Big|_{V_{\text{d}} = \bar{V}_{\text{d,final}}} \tag{19}$$

by some time function $\bar{F}(t)$. The transfer takes then place on the time interval $[t_{\text{init}}, t_{\text{final}}]$.

More precisely, we specify:

$$\langle F^\star(t) \rangle_{\text{dc}} = \begin{cases} \bar{F}_{\text{init}} & \text{for } t < t_{\text{init}} \\ \bar{F}(t) & \text{for } t \in [t_{\text{init}}, t_{\text{final}}] \\ \bar{F}_{\text{final}} & \text{for } t > t_{\text{final}} \end{cases} \tag{20}$$

This procedure grants the possibility to plan in an off-line manner the nominal trajectory $i^\star(t)$ for the remaining state variable i and the nominal average control input $u^\star_{\text{av}}(t)$. The nominal trajectory planning for the state variable i is intimately related to the initial and final values of the average steady state total stored energy that in this case is the flat output of the system. The off-line planned derivative of the total energy (12) induces a corresponding nominal trajectory for the unity power factor line current amplitude $A(t)$, as follows:

$$A(t) = \frac{2}{E}\left\langle \dot{F}^\star(t) \right\rangle_{\text{dc}} + \frac{2}{E R_{\text{L}}}\left\langle (v^\star(t))^2 \right\rangle_{\text{dc}} \tag{21}$$

with

$$\langle v^\star(t) \rangle_{\text{dc}} = \begin{cases} \bar{V}_{\text{d,init}} & \text{for } t < t_{\text{init}} \\ \bar{V}(t) & \text{for } t \in [t_{\text{init}}, t_{\text{final}}] \\ \bar{V}_{\text{d,final}} & \text{for } t > t_{\text{final}} \end{cases} \tag{22}$$

where $v^\star(t)$ is the rest-to-rest desired trajectory for the average output voltage of the rectifier.

Thus, it follows that

$$i^\star(t) = A(t)\sin(\omega t)\,. \tag{23}$$

with amplitude $A(t)$ according to equation (21). The nominal average control input $u^\star_{\text{av}}(t)$ is fully compatible with the line inductor current reference $i^\star(t)$.

Differentiating equation (23) with respect to time together with (21) and insertion in (9) leads to the time-varying nominal average control

$$\begin{aligned} u^\star_{\text{av}}(t) &= \frac{1}{v^\star(t)}\Big(E\sin(\omega t) - L\frac{d}{dt}i^\star(t)\Big) \\ &= \frac{1}{v^\star(t)}\Bigg(\sin(\omega t)\Big(E - \frac{2L}{E}\ddot{F}^\star(t) - \frac{4L}{E R_{\text{L}}}v^\star(t)\,\dot{v}^\star(t)\Big) \\ &\quad - \cos(\omega t)A(t)\,L\omega\Bigg) \end{aligned} \tag{24}$$

in which we use the planned trajectories (20) and (22), in a slight abuse of notation.

IV. Proposed Controller

The proposed reference trajectory is stabilized by an exact tracking error dynamics passive output feedback controller, as studied in [6], [7], [8].

In the particular case of the boost-based unity power factor rectifier, the tracking error dynamics results from subtracting the reference dynamics, expressed in (9)-(10), from the system dynamics equations, as given by (3)-(4). The error dynamics reads:

$$L\,\dot{e}_i = -u_{\text{av}}\,e_v - v^\star(t)\,e_u \tag{25}$$

$$C\,\dot{e}_v = u_{\text{av}}\,e_i - \frac{1}{R_{\text{L}}}e_v + i^\star(t)\,e_u \tag{26}$$

with the tracking errors $e_i = i(t) - i^\star(t)$, $e_v = v(t) - v^\star(t)$ and $e_u = u_{\text{av}}(t) - u^\star_{\text{av}}(t)$.

Following [6], [7], [8], we propose the linear time-varying control feedback control law

$$e_u = -\gamma\left(-v^\star(t)\,e_i + i^\star(t)\,e_v\right) \tag{27}$$

with positive real constant γ. This control law guarantees asymptotic tracking as may be shown employing the positive definite Lyapunov-function

$$V(e) = \frac{1}{2}(e_i, e_v)\begin{pmatrix} L & 0 \\ 0 & C \end{pmatrix}\begin{pmatrix} e_i \\ e_v \end{pmatrix}\,.$$

For uniform asymptotic stabilization of the tracking error $e = 0$, we may check negative definiteness of $\dot{V} = \frac{\partial V}{\partial e}\dot{e}$ for all times, which means to verify that the *dissipation matching condition* is satisfied. This condition takes the form

$$\begin{bmatrix} \gamma\,(v^\star(t))^2 & -\gamma\,i^\star(t)v^\star(t) \\ -\gamma\,i^\star(t)v^\star(t) & \frac{1}{R_{\text{L}}} + \gamma\,(i^\star(t))^2 \end{bmatrix} > 0$$

and positive definiteness of this matrix is given uniformly for any non-zero voltage trajectory. This property may easily be guaranteed within the voltage planning, see equation (22).

Finally, the average linear time-varying controller for the unity-power-factor rectifier that assures uniform asymptotic tracking reads:

$$\begin{aligned} u_{\text{av}}(t) &= u^\star_{\text{av}}(t) + \gamma\left(v^\star\,(i - i^\star) - i^\star\,(v - v^\star)\right) \\ &= u^\star_{\text{av}}(t) + \gamma\left(v^\star(t)\,i(t) - i^\star(t)\,v(t)\right) \end{aligned} \tag{28}$$

with controller gain γ chosen positive.

V. Experimental Results

The performance of the passive output feedback control scheme for the *boost type unity power factor rectifier* is verified by experimental results. Fig. 2 shows the experimental circuit diagram and Tab. I corresponds to the specifications used in the setup. An experimental 95 VA single-phase rectifier is employed to operate as a power factor corrector. The continuous realtime control was programmed via Matlab/Simulink and subsequently downloaded on a data acquisition card (DAQCard-6062E)

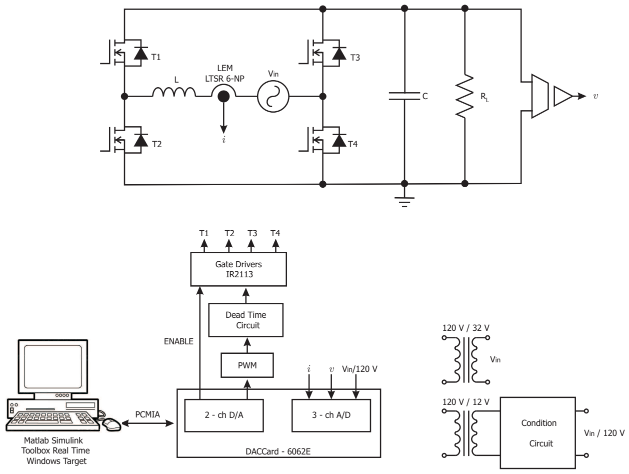

Fig. 2. The boost type unity power factor rectifier circuit

TABLE I
SPECIFICATIONS

Power rating	95 VA
Supply voltage	120/32 V
Switching device	IRF640 (200 V/18A)
Commutation frequency	$45\,kHz$
Power diode	MBRF20100CT
Acquisition device	DAQCard-6062E
DC capacitor	$C = 1000\mu$F
Boost inductor	$L = 1$mH
Resistor load	$R_{\mathrm{L}} = 300\,\Omega$
Sampling time	$200\,\mu\,\mathrm{sec}$
Control method	Passivity-based

that established the communication between the computer and the circuit's measurements and modulation.

In view of (22), the desired voltage trajectory $\bar{V}(t)$ was specified in style of an interpolating Bézier polynomial of 10th order. The initial and final value was set to $\bar{V}_{\mathrm{d,init}} = 44\,V$ and $\bar{V}_{\mathrm{d,final}} = 85\,V$, to be taken at the initial time instant $t_{\mathrm{init}} = 0.5\,\mathrm{sec}$ and the final time instant $t_{\mathrm{final}} = 1\,\mathrm{sec}$, respectively.

Consequently, we specified the reference trajectory

$$\bar{V}(t) = \bar{V}_{\mathrm{d,init}} + \left(\bar{V}_{\mathrm{d,final}} - \bar{V}_{\mathrm{d,init}}\right) b\left(\frac{t - t_{\mathrm{init}}}{t_{\mathrm{final}} - t_{\mathrm{init}}}\right) \quad (29)$$

with the Bézier polynomial

$$\begin{aligned} b(\tau) = {} & 252\,\tau^5 - 1050\,\tau^6 + 1800\,\tau^7 - 1575\,\tau^8 \\ & + 700\,\tau^9 - 126\,\tau^{10}\,. \end{aligned} \quad (30)$$

Accordingly, the flat output reference trajectory is planned. That is, we resort to equation (20) and use the Bézier polynomial (30) again.

Fig. 3 shows the corresponding closed-loop output voltage response for the desired output voltage as specified above when changing the stationary regimes from time instant $t_{\mathrm{init}} = 0.5\,\mathrm{sec}$ to $t_{\mathrm{final}} = 1\,\mathrm{sec}$. Fig. 4 depicts the closed-loop response of the input current for this setting. Initially, the current amplitude was $A(0.5\,\mathrm{sec}) = 0.3\,\mathrm{A}$ and $A(1\,\mathrm{sec}) = 1.15\,\mathrm{A}$, finally. Consequently, the initial

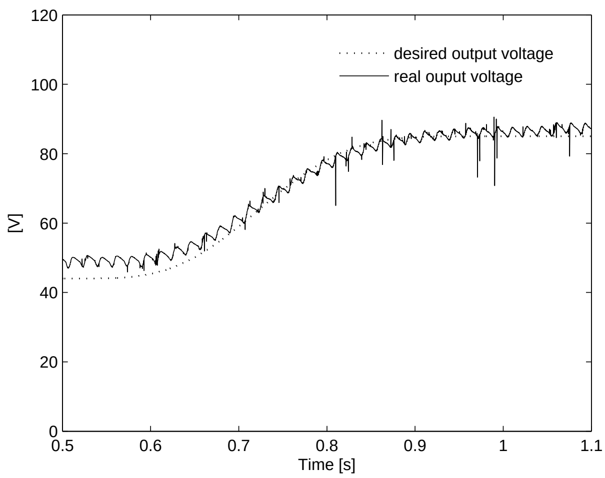

Fig. 3. Measured output voltage and reference output voltage

current signal was $i[0.5\,\mathrm{sec}] = 0.3\,\sin(\omega t)$ and $i[1\,\mathrm{sec}] = 1.15\,\sin(\omega t)$, finally. The distortions stem from the line-voltage notching and distortion, as well as from the high-frequent commutations of the semiconductors, see [9]. Furthermore, Fig. 4 reveals that the supply voltage signal is in phase with the input current signal of the system. For a better illustration, the desired and the real input current signals were multiplied by a factor of 10 and the start of the time line was set to where already considerable amplifications are visible. Fig. 5 shows the respective

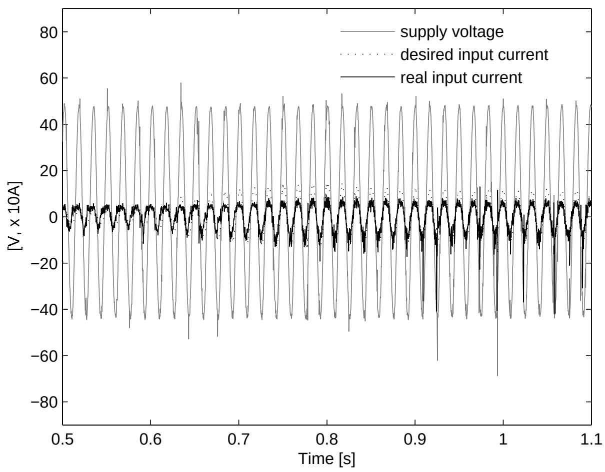

Fig. 4. Measured supply voltage, measured input current, and reference input current—measured voltage and current are in phase

nominal and real ac control input, which remains in the physical bounds of $[-1, 1]$. Fig. 6 and Fig. 7 depict the power factor and the power distribution, respectively, during the desired output voltage transition shown in Fig. 3.

VI. CONCLUSION

In this article, we have devised a controller for a power factor pre-compensator of the boost type. We control the tracking of the output voltage at a resistive

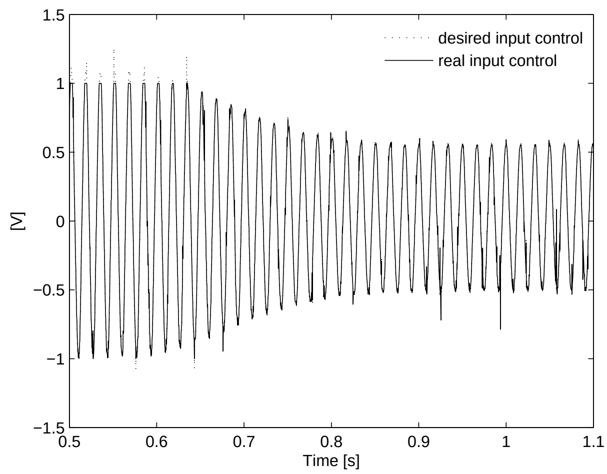

Fig. 5. Average control input and nominal control input

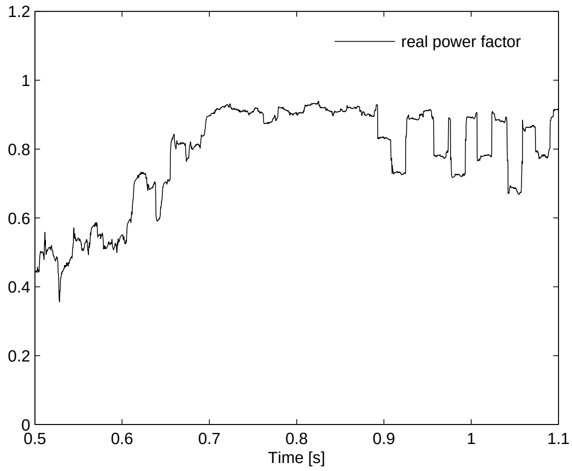

Fig. 6. Power factor during transition

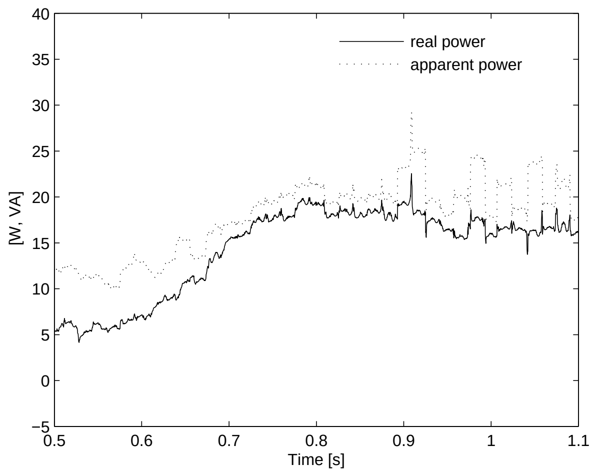

Fig. 7. Real power and apparent power during transition

load of the circuit in such a manner that the source voltage and the input current are both in phase, that is, the circuit nominally exhibits a unity power factor. The nominal, feedforward control is computed exploiting the flatness property of the average model of system. The voltage transition is planned between two stationary regimes with respect to the desired DC components of the voltages resorting to polynomials of Bézier type. In the same style, the transition between the corresponding energy levels (flat output) is planned. All these steps may be carried out off-line. For the feedback control part that may compensate for perturbations, an already introduced exact tracking error dynamics static passive output feedback control policy is employed. The overall control policy may be regarded as a linear time-varying average controller. Practically, the control is implemented resorting to the usual high frequency switching in a pulse width modulation scheme, provided by a data acquisition system. The experimental results underscore that the controlled tracking performance of the output voltage is quite satisfactory, even under unity power factor requirements.

Further studies may include the online-adaption to changes of uncertain parameters and to changes in resistive loads. An other issue is to strive for a reduction of sensors. The latter entitles the need for observation schemes that are robust with respect to substantial levels of noise on the measurement signals.

VII. Acknowledgments

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