FLATNESS-BASED FAULT TOLERANT CONTROL OF A NONLINEAR MIMO SYSTEM USING ALGEBRAIC DERIVATIVE ESTIMATION

P. Mai * , **C. Join** ** , **J. Reger** ***

* *Institute of Automation and Control (EIT 8.1), Universität der Bundeswehr München, Werner-Heisenberg-Weg 39, 85579 Neubiberg, Germany (e-mail: pmai@ieee.org)*
** *CRAN UMR 7039 CNRS - Nancy Université & ALIEN (INRIA-FUTURS project), BP 239, 54506 Vandœuvre-lès-Nancy, France (e-mail: cedric.join@cran.uhp-nancy.fr)*
*** *EECS Control Laboratory, University of Michigan, USA (e-mail: reger@ieee.org)*

Abstract: A flatness-based approach to fault tolerant control is proposed. The approach uses the recently published algebraic derivative estimation method for the estimation of those output derivatives that are necessary for determining intermittent actuator faults. The rapid performance of the estimation allows for an accommodation of the control to the fault. Additionally, taking into account the control saturations a novel classification scheme for actuator faults is introduced that exhibits a comprehensible graphical representation in terms of reachable sets. Dependent on the respective class of fault, an online adaptation of the reference trajectory is carried out. The ideas are demonstrated on a nonlinear MIMO system, which corresponds to an underactuated rigid body.

Keywords: Fault tolerant systems, nonlinear control systems, estimation.

1. INTRODUCTION

In recent years, methods for model-based fault diagnosis have experienced increasing attention. This may be due to a couple of novel perspectives and techniques for state and parameter estimation of nonlinear systems that significantly contributed to enlarge the capabilities for detection and treatment of occurring faults out of model-data, only (see for example (Chen, 1999; De Persis and Isidori, 2001; Gertler, 1998; Frank, 1990; Staroswiecki and Comtet-Varga, 2001)). In this paper, which is the second of a series of papers on this topic, we focus on the *Fundamental Problem of Residual Generation* (FPRG), firstly introduced by (Jones, 1973), which aims at the fault detection and isolation in the case of multiple faults. This problem has been dealt with using approaches of linear and nonlinear type.

Concerning linear methods, a vast literature underscores the advanced levels that have been achieved in this field of research. Among all the available literature we would like to single out the ideas in (White and Speyer, 1987; Massoumnia, 1989) which involve the design of detection filters that are based on a geometric setting. Refraining from the use of filters and/or observers, an algebraic approach was taken in (Fliess *et al.*, 2004).

Fault diagnosis for nonlinear systems has been concerned in (Chen, 1999; De Persis and Isidori, 2001; Gertler, 1998; Join, 2002; Join *et al.*, 2003), for instance. In the field of fault tolerant control, recently, encouraging results were obtained applying algebraic techniques within the control of a three tank system (Fliess *et al.*, 2005b). In these approaches, it is common practice to modify the control law by taking into account the result of the fault diagnosis procedure in order to reduce the effect of this fault on the control activity.

Usually, this leads to an additive control action, see for instance (Lunze *et al.*, 2001; Theilliol *et al.*, 2002). However, the saturation limits of the control signal, always present in practice, may require to change online the reference trajectory in order to keep up a sucessful fault accommodation scheme (Tarbouriech and Garcia, 1997; Kapila and Grigoriadis, 2002). Meeting this particular requirement is one of the main contributions of this work.

In this paper, some ideas firstly published in (Mai *et al.*, 2006) are extended and generalized to a nonlinear MIMO system: a third order system desribing the movement of a rigid body. This example gives evidence for the general applicability of our methods to nonlinear differential flat systems of any kind.

The paper is organized as follows: Firstly, the algebraic derivative estimation method is briefly recalled in Section 2. Afterwards in Section 3, the system model and its flatness-based trajectory planning is explained. The main contributions of the paper are given in Sections 4, 5 and 6. A fault estimation and accommodation strategy is proposed which naturally leads to a fault classification scheme that may be graphically represented in a clear manner. Finally, an online trajectory replanning procedure is suggested that is based on the result of the previous fault classification. Section 7 is devoted to the conclusions and describes some forthcoming works.

2. ALGEBRAIC DERIVATIVE ESTIMATION

We use a recently published derivative estimation method (Fliess *et al.*, 2004; Fliess *et al.*, 2005a; Mboup *et al.*, 2007) [1] which is based on differential algebraic manipulations of a polynomial function of time.

In light of this method, an estimate $\hat{\dot{x}}(t)$ for the first time derivative of a noisy measurement signal $y(t) = x(t) + n(t)$, with $x(t)$ as the uncorrupted original signal and $n(t)$ an additive noise signal, may be found by

$$\hat{\dot{x}}(t) = -\frac{6}{T_{\text{win}}^3} \int_0^{T_{\text{win}}} (2\sigma - T_{\text{win}}) y(t-\sigma) d\sigma, \quad (1)$$

where T_{win} is an integration window length ($T_{\text{win}} = 0.1$, chosen here). A reset and repeated integration of the time window permits to estimate the derivative at any sampled instant of time t.

The time integral in (1) shows low pass filter characteristics, capable of attenuating noises. Here, the noise on $y(t)$ is viewed as a highly fluctuating, or oscillatory, phenomenon [2].

[1] See theses references for more details and the methodological background.

[2] A precise mathematical foundation can be found in (Fliess, 2006).

3. SYSTEM REPRESENTATION AND FLATNESS BASED TRAJECTORY PLANNING

In this section, we present a flat nonlinear system which serves as a demonstration system for our considerations. Briefly, we describe an offline trajectory planning procedure that is based on a differential parametrization. Consider the system

$$\begin{aligned} 0.5\,\dot{x}_1 &= -0.15\,x_2 x_3 + u_1 + f_{\text{a},1} \\ 0.4\,\dot{x}_2 &= 0.05\,x_1 x_3 + u_2 + f_{\text{a},2} \\ 0.55\,\dot{x}_3 &= 0.1\,x_1 x_2 \end{aligned} \quad (2)$$

which may be seen as an underactuated rigid body (see (Sira-Ramírez *et. al*, 2004)), with angular velocities x_i, $i = 1, 2, 3$, of the body around its i-th principal axis of rotation and control inputs u_1, u_2. The quantities $f_{\text{a},1}$, $f_{\text{a},2}$ are unknown additive actuator faults. The fault-free system, that is $f_{\text{a},1} = 0$ and $f_{\text{a},2} = 0$, is differentially flat (Fliess *et al.*, 1995) with flat outputs $F_1 = x_1$ and $F_2 = x_3$. The differential parametrization of the system in terms of the flat outputs reads:

$$\begin{aligned} x_2 &= 5.5\,\dot{F}_2/F_1 \\ u_1 &= 0.5\,\dot{F}_1 + 0.825\dot{F}_2 F_2/F_1 \\ u_2 &= 2.2\,(\ddot{F}_2 F_1 - \dot{F}_2 \dot{F}_1)/F_1^2 - 0.05\,F_1 F_2. \end{aligned} \quad (3)$$

It is assumed that all states, F_1, F_2 and x_2, are measured directly and corrupted by measurement noise (added up within the simulations).

In this work, the key idea is to adapt the fault tolerant control actions to the limitations of the control saturations. If we define a saturation function $\text{sat}_S(x)$ by

$$\text{sat}_S(x) = \begin{cases} S & \text{for} \quad x \geq S \\ x & \text{for} \quad -S < x < S \\ -S & \text{for} \quad x \leq -S, \end{cases} \quad (4)$$

for some saturation threshold $S > 0$ then we can introduce the so called *free control inputs* $u_1'(t)$, $u_2'(t)$, which denote the actually intended control inputs if no saturation were present. Therefore, we set

$$u_i(t) = \text{sat}_{S_i}(u_i'(t)), \ i = 1, 2\,. \quad (5)$$

We will restrict the further analysis to steplike actuator faults since in this case the future dynamics of the system can easily be predicted, once the amplitude of the fault has been estimated. Let $T_{\text{a},i}$ denote the time instant when the actuator fault occurs at actuator i, $i = 1, 2$, and $F_{\text{a},i}$ the amplitude, then $f_{\text{a},i}(t)$ is defined by

$$f_{\text{a},i} = F_{\text{a},i}\sigma(t - T_{\text{a},i}), i = 1, 2\,, \quad (6)$$

where $\sigma(t)$ denotes the unit step.

Based on the system's differential parametrization (3) the offline trajectory planning problem is an easy task. Let $F_1^\star(t)$ and $F_2^\star(t)$ be the reference trajectories for the flat outputs $F_1(t)$ and $F_2(t)$ that we would like to track. Let $u_1^\star(t)$ and $u_2^\star(t)$ be the corresponding nominal control inputs which are given by

$$u_1^\star = 0.5\,\dot{F}_1^\star + 0.825\,\dot{F}_2^\star F_2^\star / F_1^\star \tag{7}$$

$$u_2^\star = 2.2\,(\ddot{F}_2^\star F_1^\star - \dot{F}_2^\star \dot{F}_1^\star)/(F_1^\star)^2 - 0.05 F_1^\star F_2^\star. \tag{8}$$

Throughout this work, we focus on keeping the free control commands $u_1'(t)$ or $u_2'(t)$ away from the saturation, since the stability analysis can prove quite difficult in this case. Let $\epsilon_i > 0$, $i = 1, 2$, be some security margins that we would like to keep the free control signals $u_i'(t)$ additionally away from their respective saturation limits S_i, and which provides a means for the controller to deal with unkown initial conditions and actuator faults. Then it is clear that $F_1^\star(t)$ and $F_2^\star(t)$ should be chosen such that

$$u_i^\star(t) \in [-S_i + \epsilon_i, S_i - \epsilon_i], \forall t \geq 0\,,\; i = 1, 2$$

which can easily be verified numerically with the help of (7) and (8), once the analytic expression for the reference trajectories $F_1^\star(t)$ and $F_2^\star(t)$ is specified. To this end, a fourth order Bézier polynomial was chosen as reference trajectory, which reads

$$F_i^\star(t) = F_i(0) + (F_i(T_\mathrm{f}) - F_i(0))\,(t/T_\mathrm{f})^2 \times \left(6 - 8(t/T_\mathrm{f}) + 3(t/T_\mathrm{f})^2\right),\; t \leq 0 \leq T_\mathrm{f}, \tag{9}$$

with $i = 1, 2$, where, obviously $F_i^\star(t) = F_i(T_\mathrm{f})$ for $t > T_\mathrm{f}$. Here, T_f denotes the system transfer time between two stationary regimes, and $F_i(0)$ and $F_i(T_\mathrm{f})$ denote the corresponding inital and stationary value of the reference trajectory.

4. FAULT IDENTIFICATION AND FAULT TOLERANT CONTROL

We now want to set up a fault tolerant flatness-based controller that guarantees asymptotically stable tracking of the system ouputs F_1 and F_2 to the reference trajectories $F_1^\star$, $F_2^\star$. In light of this, we assume that the free control signals $u_1'(t)$ and $u_2'(t)$ will not be cut by their respective saturation blocks. Fault tolerant behavior of the controllers, in this case, is achieved by adding a negative estimate of the actuator fault on the control expressions of the fault-free case. Note that this strategy is viable for treating additive as well as multiplicative actuator faults because any multiplicative actuator fault may be written as an additive fault. Also, the restriction to steplike actuator faults is not obligatory for this accommodation scheme, though it is done here to set a compatible basis for the further sections.

We may obtain estimates for the actuator faults, $\hat{f}_{\mathrm{a},1}(t)$ and $\hat{f}_{\mathrm{a},2}(t)$, from the system dynamics (2), that is

$$\hat{f}_{\mathrm{a},1} = 0.5\,\hat{\dot{x}}_1 + 0.15\,x_2 x_3 - u_1 \tag{10}$$

$$\hat{f}_{\mathrm{a},2} = 0.4\,\hat{\dot{x}}_2 - 0.05\,x_1 x_3 - u_2 \tag{11}$$

where $\hat{\dot{x}}_i$, $i = 1, 2$, denotes the estimate of the first derivative of the state x_i that can be measured subject to noise, only. For this purpose, the algebraic derivative estimation scheme was used. From the differential parametrization (3) of the system, which is valid for the fault-free case, a flatness-based tracking control law can be obtained when replacing $\dot{F}_1$ and $\ddot{F}_2$ by the auxiliary control inputs v_1 and v_2, hence

$$\dot{F}_1 = v_1,\;\; \ddot{F}_2 = v_2, \tag{12}$$

which renders the system locally linear and decoupled. Thus, specifying

$$v_1 = \dot{F}_1^\star + K_1(F_1^\star - F_1) \tag{13}$$

$$v_2 = \ddot{F}_2^\star + K_2(\dot{F}_2^\star - \dot{F}_2) + K_3(F_2^\star - F_2) \tag{14}$$

makes the tracking errors $e_1(t) = F_1(t) - F_1^\star(t)$ and $e_2(t) = F_2(t) - F_2^\star(t)$ asymptotically converge to zero for an appropriate choice of control gains K_1, K_2, K_3.

Note that the implementation of the control law (14) requires the first derivative of the flat output F_2. Since only noisy measurements of F_1 and F_2 are available, the algebraic derivative estimation method was used once more to obtain $\hat{\dot{F}}_2$. Therefore, the control laws (13) and (14) finally read

$$\begin{aligned} v_1 &= \dot{F}_1^\star + K_1(F_1^\star - F_1) \\ v_2 &= \ddot{F}_2^\star + K_2(\dot{F}_2^\star - \hat{\dot{F}}_2) + K_3(F_2^\star - F_2) \\ u_1' &= 0.5 v_1 + 0.825\,\hat{\dot{F}}_2 F_2 / F_1 - \hat{f}_{\mathrm{a},1} \\ u_2' &= 2.2\,(v_2 F_1 - \hat{\dot{F}}_2 \hat{\dot{F}}_1)/(F_1)^2 - 0.05\,F_1 F_2 - \hat{f}_{\mathrm{a},2}. \end{aligned} \tag{15}$$

The system will now show an asympotically stable tracking when the following conditions are met:

(1) $\hat{f}_{\mathrm{a},i} \approx f_{\mathrm{a},i}$, $i = 1, 2$.
(2) The saturations will not be hit, that is: $u_i'(t) \in [-S_i, S_i], \forall t \geq 0$, $i = 1, 2$.

The first condition only depends on the convergence rate of the derivative estimation method and is fulfilled for slowly varying actuator faults. Especially, for step-like actuator faults, which are defined by (6), the amplitude F_a is perfectly estimated after a very small duration. The second condition may be violated whenever the actuator faults show amplitudes which cannot be compensated due to the control saturation limits. This problem is adressed in the next sections.

The following simulation results show the efficiency of the proposed methods. The reference trajectories were chosen according to (9). Two actuator faults were injected; plotted in Fig. 1 together with the respective estimations. The results are quite accurate in spite of measurement noise which was chosen equally distributed with a maximum amplitude of ± 0.01. Reference trajectories and system outputs are depicted in Fig. 3. The performance of the asymptotic tracking is impressive, despite of the high amplitude of the

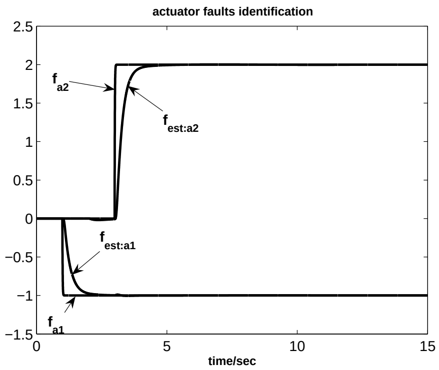

Fig. 1. Comparison of injected actuator faults $f_{a,1}, f_{a,2}$ and estimates $f_{est,a,1}, f_{est,a,2}$.

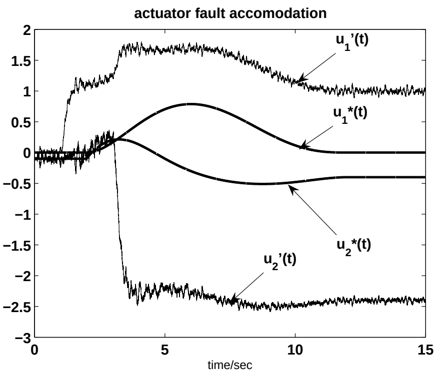

Fig. 2. Reference control signals $u_1^\star, u_2^\star$ and free control signals u_1', u_2'.

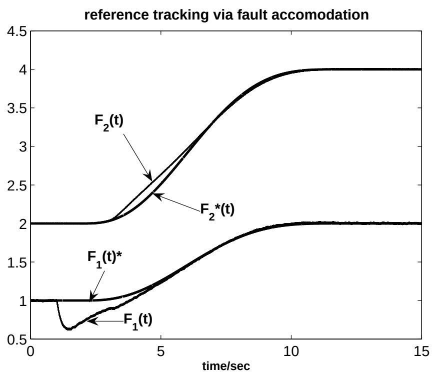

Fig. 3. Comparison of reference trajectories $F_1^\star$, $F_2^\star$ and measured system outputs F_1, F_2.

actuator faults and of the lack of an integral term in the control law. Finally, Fig. 2 shows the reference control signals together with the free control signals which are shifted by the negative value of the resp. estimated actuator fault. The control saturations where chosen such that no cutting of the free control signals occurs in this example.

5. GRAPHICAL REPRESENTATION OF REACHABLE REGIONS AND ACTUATOR FAULT CLASSIFICATION

We now use a graphical display of the so-called reachable regions, by which we understand the 2-dimensional set of stationary points $(F_1^\star(T_f), F_2^\star(T_f))$ that can be chosen as the final values of the reference trajectories $F_1^\star(t)$, $F_2^\star(t)$, once the analytic form of the reference trajectories is fixed. This graphical representation will prove quite useful for introducing a new classification of actuator faults, which may be considered very valuable by the system operator.

For explaining the idea of a *reachable region* consider the offline trajectory planning problem before the system is switched on. Assume for simplicity that the initial conditions of the system match the initial values of the reference trajectories and that no actuator faults occur. As a consequence, we will observe perfect tracking, that is, $F_1(t) = F_1^\star(t)$, $F_2(t) = F_2^\star(t)$ as long as the nominal control signals $u_i^\star(t)$, $i = 1, 2$, reside fully within their respective saturation intervals $[-S_i, S_i]$; in this case, $u_i' = u_i^\star(t)$, $i = 1, 2$, is valid. For a fixed starting point $(F_1^\star(0), F_2^\star(0))$, a fixed analytic form of the reference trajectory—in our case the Bézier polynomial defined by (9)—and a fixed transfer time T_f, it is now straightforward to vary the designated arrival point $(F_1^\star(T_f), F_2^\star(T_f))$ and check wether $u_i^\star(t) \in [-S_i+\epsilon_i, S_i-\epsilon_i], \forall t \geq 0$, $i = 1, 2$ is valid, where $u_i^\star(t)$ is given by (7) and (8). The set of all points $(F_1^\star(T_f), F_2^\star(T_f))$ that satisfy $u_i^\star(t) \in [-S_i + \epsilon_i, S_i - \epsilon_i], \forall t \geq 0$, $i = 1, 2$ defines the *reachable region*. An exemplary result is depicted in Fig. 4.

For the fault classification scheme based on the graphical representation of reachable regions, it is important to recall that any steplike actuator fault $f_{a,i}$ of amplitude $F_{a,i}$ leads to a shift of $-F_{a,i}$ with respect to the free control signal $u_i'(t)$ for the fault-free case. This is equivalent to the statement that no actuator fault actually occurred, but the saturation limits of the free control command were shifted by $+F_{a,i}$, that is $u_i \in [-S_i+F_{a,i}, S_i+F_{a,i}]$. Therefore, the calculation of reachable regions during system operation after the instant when actuator faults have been identified can be done in the same way as in the fault-free case, just the valid control intervals have to be adapted.

The size of the reachable region, in general, will strongly depend on the chosen transfer time T_f. Heuristically speaking, a larger region can be reached for a greater transfer time, given fixed control saturations. The following fault classification is now proposed:

(1) **Accommodable Fault:** The control law can be accommodated to the actuator fault

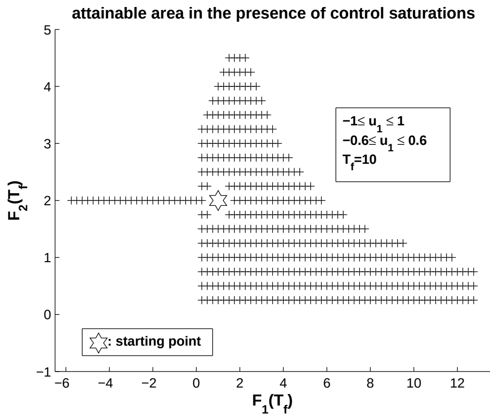

Fig. 4. Reachable region $(F_1(T_\mathrm{f}), F_2(T_\mathrm{f}))$ for $T_\mathrm{f} = 10$, $u_1 \in [-1, 1]$, $u_2 \in [-0.6, 0.6]$ and starting point $F_1(0) = 1, F_2(0) = 2$.

according to (15), thus $u'_i(t) \in [-S_i, S_i], \forall t \geq 0$, $i = 1, 2$ is valid.

(2) **Dynamically Severe Fault:** The accommodation of the control to the actuator fault according to (15) without having any saturation hits can only be done when the original transfer time T_f of the system trajectory is enlarged, while the same stationary point $(F_1^\star(T_\mathrm{f}), F_2^\star(T_\mathrm{f}))$ can still be reached.

(3) **Severe Fault:** The originally designated stationary arrival point $(F_1^\star(T_\mathrm{f}), F_2^\star(T_\mathrm{f}))$ cannot be reached anymore without having a saturation hit of the free control inputs (15). A new stationary point must be chosen.

Fig. 5 illustrates the different fault classes. It shows the growth of the reachable regions when the transfer time T_f is enlarged. Here, the boundary of the reachable regions were calculated for each T_f. System parameters were chosen as in Fig. 4. If the originally designated transfer time $T_{\mathrm{f,des}}$ was set to be 6 sec and a maximum transfer time of 15 sec is fixed then the two red points that represent two intended stationary points represent a situation with a Dynamically Severe Fault and a Severe Fault: in the first case, the red point can still be reached, though in a larger transfer time of $6\,\mathrm{sec} < T_\mathrm{f} < 9\,\mathrm{sec}$, whereas in the second case the red point will not be reachable within the maximum time of 15 sec. Consequently, an other stationary point will have to be chosen.

6. DYNAMIC TRAJECTORY REPLANNING

In this section, we briefly describe an online trajectory adaptation scheme (for another example see (Devos and Lévine, 2006)) that might succeed the fault classification pattern of the last section. Note that the flatness of the system gives us entire knowledge of the future dynamics even in the case of actuator faults, provided that the free control

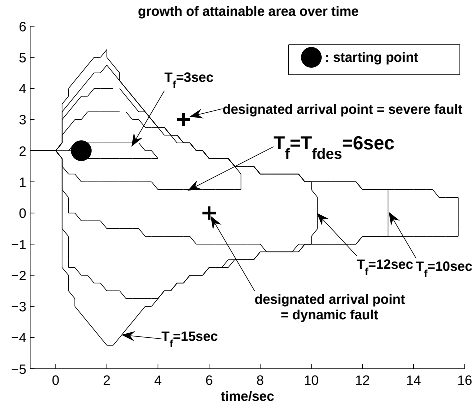

Fig. 5. Development of reachable region when the transfer time T_f is varied, for $u_1 \in [-1, 1]$, $u_2 \in [-0.6, 0.6]$ and initial condition $F_1(0) = 1, F_2(0) = 2$. The originally desired arrival time was $T_{\mathrm{f,des}} = 6\,\mathrm{sec}$.

signals $u'_1(t)$, $u'_2(t)$ never hit their saturation limits. It is therefore intuitive to act in the following way once an actuator fault has been classified:

- In the case of an accommodable fault, the original trajectory can be kept.
- In the case of a dynamically severe fault, it is sufficient to enlarge the arrival time of the system, while keeping the same stationary values.
- In the case of a severe fault, it is necessary to modify the stationary values, and in general, the arrival time as well, according to a prespecified metric that depends on the real application.

The dynamic trajectory adaptation is demonstrated in Fig. 6. As shown in Fig. 7, the adaptation proved necessary in order to avoid a saturation hit of u_1.

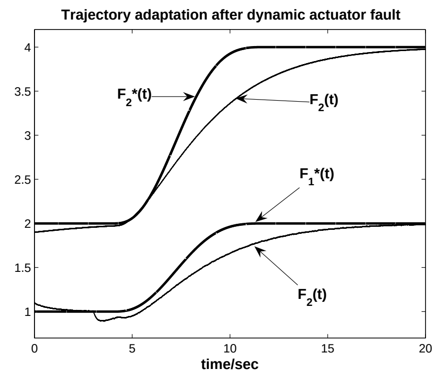

Fig. 6. Postponing the arrival time after the occurrence of a dynamically severe actuator fault. The system outputs F_1,F_2 and the adapted reference trajectories perfectly coincide right after trajectory adaptation. $F_1^\star, F_2^\star$ denote the original reference trajectories.

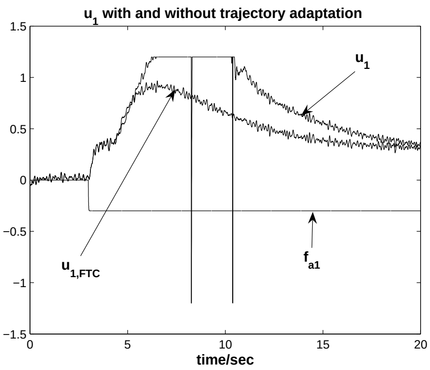

Fig. 7. Demonstration of how the dynamic adaptation of the reference trajectory avoids the saturation hit of $u_{1,\mathrm{FTC}}$, whereas the control u_1 gets severely cut by the saturation when no trajectory adaptation is applied. The actuator fault $f_{\mathrm{a},1}$ is given as well.

7. CONCLUSIONS

A novel approach to fault diagnosis and fault accommodation is proposed. To this end, we combine a new fault estimation method, which is based on derivative estimations of measured time signals, with the notion of flatness-based control so as to ensure actuator accommodation to the fault, even in a setting with control saturations. Furthermore, this work gives, probably for the first time, a precise mathematical characterization of different types of faults which are of practical relevance (accommodable fault, dynamically severe fault, severe fault), enhancing heuristic definitions as in (Isermann, 1984) for example. Future work will focus on the fault tolerant control of uncertain nonlinear systems.

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