Load Torque Estimation and Passivity-Based Control of a Boost-Converter/DC-Motor Combination

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***Abstract*—An algebraic approach is presented for the fast feed-forward adaptation of the angular velocity trajectory tracking task in a Boost-converter driven dc-motor system. For the adaptation, the load torque perturbations are assumed piecewise constant and are, nonasymptotically, online estimated using the available noisy measurements of the state variables. The controller is a linear controller based on the exact tracking error dynamics passive output feedback (ETEDPOF) controller design methodology including suitable adaptive feed-forward precompensation depending explicitly on the precisely estimated torque. The performance of the adaptation, which is achieved by means of the algebraic online-estimation of the current unknown load torque, is successfully validated in an experimental laboratory setup.**

***Index Terms*—Algebraic estimation, electrical drives, flatness-based trajectory planning, passivity-based control, power electronics.**

I. Introduction

IN MANY industrial applications, dc-motors serve as simple devices for implementing velocity tracking tasks. Feedback control schemes, typically, are of the proportional–integral (PI) and proportional–integral–differential (PID)-type (for example, see [10] and [17]). Usually, the voltage for the speed control is supplied by thyristor-based phase controlled rectifiers [18] operated at higher switching frequencies (above 40 kHz) [12] and are designed to provide a continuous armature current under various load situations [10].

The robustness problem with respect to load changes on the dc-motor, is frequently addressed by an online asymptotic torque estimation through an observer (see Chiasson [2] and Hagenmeyer *et al.* [7]). In this brief, we use an alternative for the torque estimation. This alternative is a so-called algebraic estimation scheme that provides accurate and fast estimation results, even when subject to noise levels encountered in practice (the theoretical features, and some of the applications, of the algebraic approach to parameter and signals estimation may be found in the work of Fliess and Sira-Ramírez [4], [5]). In the linear case, the estimation of unknown parameters is guaranteed in an online fashion through the evaluation of a simple time-varying expression resorting to integrations of inputs and outputs alone.

In this paper, we present a smooth "accelerator", or "braking", control system for a dc-motor constituted by a power converter of the Boost type. The main task is to achieve angular velocity regulation of the motor shaft under nonconstant loads; for simplicity, stepwise changing constant loads are assumed here. A simple *linear time-varying* state feedback controller, based on exact tracking error dynamics passive output feedback (ETEDPOF) strategy is shown to stabilize the state trajectory tracking error to zero while requiring measurements of the converter current and voltage, only. For fundamental generalities about the ETEDPOF strategy see: Sira-Ramírez, [20]. Numerous applications of the technique were developed in H. Sira-Ramírez and Silva-Ortigoza [22] in connection with traditional power electronics and, finally, for many applications dealing with other interesting combinations of dc-power converters and dc-motors the reader is refered to the works by J. Linares-Flores *et al.* [13]–[15].

The dynamic average model of the Boost-converter/dc-motor combination is shown to conform to a special energy managing structure which is specially suitable for the ETEDPOF controller design methodology, which, incidentally, is intimately related to traditional static passivity-based output feedback controller design techniques. In contrast to dynamic output feedback approaches with precompensation (see [1]), the ETEDPOF controller, is not only linear but also static.

The required reference signal of the converter is generated exploiting a partial differential flatness property of the considered tandem system (*differential flatness* was introduced by Fliess *et al.* in [6]. The subject has undergone considerable development ever since. For a recent book, see: Sira-Ramírez and Agrawal [21]). The corresponding (nominal) reference signals for the average control and the converter input current are derived from complementary stored energy considerations in the vicinity of stationary regimes. These reference expressions are seen to include an estimated value of the load torque parameter, which is then incorporated in the feedforward computed strategy. Consequently, the proposed policy may be used to tackle the above-stated robustness problem. As it is common practice, a polynomial of Bézier type is specified for a smooth energy interpolation trajectory corresponding to the initial and final desired angular shaft velocities. In turn, this trajectory completely defines in an offline manner the reference trajectories of the converter input current and the average control.

The organization of this brief is as follows. Section II presents the average modeling of the Boost-converter/dc-motor cascaded

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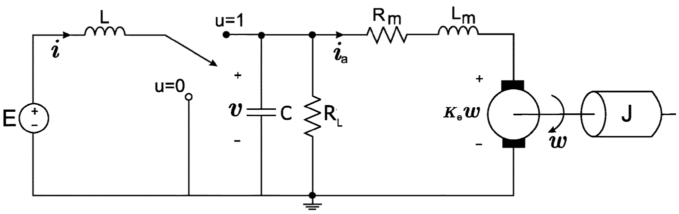

Fig. 1. Schematic of the boost-converter/dc-motor combination.

arrangement. In Section III a linear time-varying feedback controller is developed, based on the ETEDPOF controller methodology. Section IV addresses the offline trajectory generation as required by the proposed feedback controller. Section V is concerned with the design of a simple algebraic estimator for the online estimation of the load torque. Section VI briefly describes the experimental setting employed for testing the effectiveness of the proposed control method. Experimental results are provided. The conclusions are drawn in the last section.

II. Model of the Boost-Converter/ DC-Motor Combination

We consider a cascade combination of a dc-to-dc power converter of the Boost type and a dc-motor, as depicted in Fig. 1. Using the laws of Kirchhoff and Newton's second law of mechanics, we obtain the following average model of the system

$$L\frac{di}{dt} = -\,v\,u_{\rm av} + E \tag{1}$$

$$C\frac{dv}{dt} = i\,u_{\rm av} - \frac{1}{R_L}v - i_a \tag{2}$$

$$L_m\frac{di_a}{dt} = v - R_m i_a - K_e\,\omega \tag{3}$$

$$J\frac{d\omega}{dt} = K_m\,i_a - B\,\omega + \tau_L \tag{4}$$

where the denotation of the nonparametric variables is

- i input current;
- v converter output voltage or armature voltage;
- i_a armature current;
- ω angular velocity of the motor shaft;
- τ_L load torque (unknown, assumed piecewise constant);
- $u_{\rm av}$ average control input, bounded to interval $[0,1]$.

Using matrix notation, the nonlinear system (1)–(4) may be represented in the following form:

$$\dot{x}(t) = \Big(J(u_{\rm av}(t)) - R\Big)\left(\frac{\partial H(x(t))}{\partial x}\right)^T + \varepsilon \tag{5}$$

with the state vector

$$x^T(t) = (i, v, i_a, \omega) \tag{6}$$

and the quadratic Hamiltonian, that is, the total stored energy of the system

$$H(x) = \frac{1}{2}x^T M x \tag{7}$$

with respect to the matrix

$$M = \begin{pmatrix} L & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & 0 & L_m & 0 \\ 0 & 0 & 0 & J \end{pmatrix} \tag{8}$$

which is positive definite and constant. The vector ε is given by

$$\varepsilon^T = \left(\frac{E}{L}, 0, 0, \frac{\tau_L}{J}\right) \tag{9}$$

and the matrices $J(u_{\rm av})$ and R by

$$J(u_{\rm av}) = \begin{pmatrix} 0 & -\frac{1}{CL}u_{\rm av} & 0 & 0 \\ \frac{1}{CL}u_{\rm av} & 0 & -\frac{1}{CL_m} & 0 \\ 0 & \frac{1}{CL_m} & 0 & -\frac{K_e}{JL_m} \\ 0 & 0 & \frac{K_m}{JL_m} & 0 \end{pmatrix} \tag{10}$$

$$R = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_L C^2} & 0 & 0 \\ 0 & 0 & \frac{R_m}{L_m^2} & 0 \\ 0 & 0 & 0 & \frac{B}{J^2} \end{pmatrix}. \tag{11}$$

Note that for $u_{\rm av}$ arbitrary the matrix $J(u_{\rm av})$ is skew-symmetric, that is $J^T(u_{\rm av}) = -J(u_{\rm av})$, since $K_e = K_m$ due to energy conservation (power absorbed by the back emf is converted to mechanical power, e.g., see [3]). Matrix R is symmetric and positive-semidefinite, i.e., $R^T = R \geq 0$.

III. Passivity-Based Average Controller Design

It is desired to have the motor shaft track a certain angular velocity reference profile $\omega^\star(t)$ and the load torque, τ_L, to be estimated. To this end, the controller presented in [14] is generalized for the estimation of an unknown load torque.

In this regard, we assume that a state reference trajectory $x^\star(t)$ satisfies the following open-loop dynamics:

$$\dot{x}^\star(t) = \Big(J(u_{\rm av}^\star(t)) - R\Big)\left(\frac{\partial H(x^\star(t))}{\partial x^\star}\right)^T + \varepsilon^\star \tag{12}$$

where $u_{\rm av}^\star(t)$ is the reference control input corresponding to the desired state reference $x^\star(t)$ and the vector $\varepsilon^\star$ contains the online estimated, (assumed) piecewise constant torque $\hat{\tau}_L$.

How a valid state trajectory $x^\star(t)$ and the respective control input reference trajectory $u_{\rm av}^\star(t)$ may be generated from the knowledge of the desired angular velocity trajectory $\omega^\star(t)$ and from the algebraically estimated load torque $\hat{\tau}_L$ is explained in Sections IV and V, respectively.

The passivity-based control is derived upon the error system dynamics. To this end, define the deviation of the state from a computed state reference trajectory as $e(t) = x(t) - x^\star(t)$. In the same manner define the control input deviation as $e_u(t) = u_{\rm av}(t) - u_{\rm av}^\star(t)$. Let

$$H(e) = \frac{1}{2}e^T M e \tag{13}$$

be a quadratic Hamiltonian for the error system. In light of (13), it is clear that

$$\left(\frac{\partial H(e)}{\partial e}\right)^T = Me = M(x-x^\star) = \left(\frac{\partial H(x)}{\partial x}\right)^T - \left(\frac{\partial H(x^\star)}{\partial x^\star}\right)^T. \tag{14}$$

Resorting to this identity, subtracting the nominal system (12) from the actual system (5) and defining $\Xi := \varepsilon - \varepsilon^\star$, the error system is a result of the following steps:

$$\begin{aligned}\dot{e}(t) &= J(u_{\rm av})\left(\frac{\partial H(x)}{\partial x}\right)^T - R\left(\frac{\partial H(e)}{\partial e}\right)^T \\ &\quad - J(u_{\rm av}^\star)\left(\frac{\partial H(x^\star)}{\partial x^\star}\right)^T + \Xi \\ &= (J(u_{\rm av}) - R)\left(\frac{\partial H(e)}{\partial e}\right)^T + (J(u_{\rm av}) - J(u_{\rm av}^\star)) \\ &\quad \times \left(\frac{\partial H(x^\star)}{\partial x^\star}\right)^T + \Xi.\end{aligned} \tag{15}$$

In light of (10), assume that

$$J(u_{\rm av}) = J_0 + J_1\, u_{\rm av} \tag{16}$$

where J_0 and J_1 are skew-symmetric, constant matrices. Hence, it follows that

$$\dot{e}(t) = (J(u_{\rm av}) - R)\left(\frac{\partial H(e)}{\partial e}\right)^T + J_1\left(\frac{\partial H(x^\star)}{\partial x^\star}\right)^T e_u + \Xi. \tag{17}$$

After a small time δ for estimating the load torque, the estimated load torque takes the same value as the real load torque in (4) (see Section V), that is

$$\hat{\tau}_L = \tau_L \tag{18}$$

becomes valid and an immediate consequence is that

$$(\varepsilon - \varepsilon^\star) = \Xi = 0. \tag{19}$$

A natural feedback control law, defined in terms of the control input error variable e_u, which achieves asymptotic stability of the origin of the error space for the average system, may then be chosen of the form

$$e_u = -\gamma\,\frac{\partial H(x^\star)}{\partial x^\star} J_1^T \left(\frac{\partial H(e)}{\partial e}\right)^T \tag{20}$$

where the constant γ is chosen to be a strictly positive parameter. Since we assume $\Xi = 0$, the closed-loop exact tracking error dynamics evolves according to

$$\dot{e}(t) = J(u_{\rm av})\left(\frac{\partial H(e)}{\partial e}\right)^T - \tilde{R}\left(\frac{\partial H(e)}{\partial e}\right)^T \tag{21}$$

with

$$\begin{aligned}\tilde{R} &= R + \gamma\, J_1 \left(\frac{\partial H(x^\star)}{\partial x^\star}\right)^T \frac{\partial H(x^\star)}{\partial x^\star} J_1^T \\ &= \begin{pmatrix} \gamma\left(\frac{x_2^\star}{L}\right)^2 & -\gamma\frac{x_1^\star x_2^\star}{LC} & 0 & 0 \\ -\gamma\frac{x_1^\star x_2^\star}{LC} & \gamma\left(\frac{x_1^\star}{C}\right)^2 + \frac{1}{R_L C^2} & 0 & 0 \\ 0 & 0 & \frac{R_m}{L_m^2} & 0 \\ 0 & 0 & 0 & \frac{B}{J^2}\end{pmatrix}.\end{aligned} \tag{22}$$

Matrix $\tilde{R}$ is positive definite if and only if $x_2^\star \neq 0$. Since $x_1^\star(t) = i^\star(t)$ and $x_2^\star(t) = v^\star(t)$ are both strictly positive in practice, matrix $\tilde{R}$ may then be assumed positive definite. Note that $\tilde{R}_{22}$ in (22) contains a fraction of the resistor load R_L. The smaller the value of R_L the faster the tracking of the reference voltage will be (to the price of increased dissipation).

Hence with skew-symmetry of $J(u_{\rm av})$ and with $H(e)$ as specified in (13), along the trajectories of the closed-loop system it follows that

$$\begin{aligned}\dot{H}(e) &= \frac{\partial H(e)}{\partial e}\,\dot{e}(t) = \frac{\partial H(e)}{\partial e}\left(J(u_{\rm av}) - \tilde{R}\right)\left(\frac{\partial H(e)}{\partial e}\right)^T \\ &= -\,\frac{\partial H(e)}{\partial e}\,\tilde{R}\left(\frac{\partial H(e)}{\partial e}\right)^T < 0\,.\end{aligned} \tag{23}$$

Since $\tilde{R}$ is positive definite whenever $t \geq 0$ the origin of the error space is an asymptotically stable equilibrium point [23]. As the control input is bounded in $[0, 1]$, generally speaking, the result is not a global one. The stability of the origin of the error space depends on the assumption that the average control input remains within the bounds, for any initial state in practice. With respect to the nominal average control, this may be guaranteed offline by an appropriate reference trajectory specification (see Section IV). Under this assumption, we then have that

$$\lim_{t\to\infty} e(t) = 0 \quad \iff \quad \lim_{t\to\infty} x(t) = x^\star(t).$$

In terms of the converter current and voltage, referring to (20), we see that the following linear, time-varying, state feedback control law governs the motion of the boost-converter/dc-motor combination to track the reference state trajectory $x^\star(t)$ with corresponding control input reference trajectory $u_{\rm av}^\star(t)$, reads

$$\begin{aligned}u_{\rm av} &= u_{\rm av}^\star(t) + \gamma\big(v^\star(t)\,(i - i^\star(t)) - i^\star(t)\,(v - v^\star(t))\big) \\ &= u_{\rm av}^\star(t) + \gamma\big(v^\star(t)\,i - i^\star(t)\,v\big).\end{aligned} \tag{24}$$

Notice that whenever $i \to i^\star(t)$ and $v \to v^\star(t)$ then for the average control $u_{\rm av} \to u_{\rm av}^\star(t)$.

IV. Reference Trajectory Generation

In view of the nature of the derived feedback control law (24), we need to generate the voltage and current references for the boost converter circuit, i.e., $v^\star(t)$ and $i^\star(t)$. Since here our focus is on a smooth accelerator or braker, we restrict the reference profiles to smooth changes between stationary regimes. We follow the steps proposed in [14], here, taking into account the unknown load torque. For the generation of the nominal average output voltage $v^\star(t)$ of the boost-converter, which clearly coincides with the average armature circuit input voltage, and for the corresponding average armature circuit current $i_a^\star(t)$, we may use their differential parameterizations in terms of the desired angular velocity $\omega^\star(t)$ and of the estimated load torque $\hat{\tau}_L$ which we assume piecewise constant, for simplicity. Indeed, from the system model (1)–(4), simple steps of manipulation show that

$$v^\star(t) = \frac{JL_m}{K_m}\ddot{\omega}^\star(t) + \left(\frac{BL_m}{K_m} + \frac{JR_m}{K_m}\right)\dot{\omega}^\star(t) + \left(\frac{BR_m}{K_m} + K_e\right)\omega^\star(t) - \frac{R_m}{K_m}\hat{\tau}_L \tag{25}$$

$$i_a^\star(t) = \frac{J}{K_m}\dot{\omega}^\star(t) + \frac{B}{K_m}\omega^\star(t) - \frac{1}{K_m}\hat{\tau}_L. \tag{26}$$

Consequently, whenever a nominal angular shaft velocity $\omega^\star(t)$ is specified, e.g., by some Bézier polynomial of sufficient degree, the variables $v^\star(t)$ and $i^\star(t)$ depend on an up-to-date value of the estimate $\hat{\tau}_L$, only.

Under equilibrium conditions, the above parametrization establishes that for the constant angular velocities $\omega^\star(t) = \bar{\omega}$, it follows that the corresponding constant values of v and i_a, denoted, respectively, by $\bar{v}$ and $\bar{\imath}_a$, are given by

$$\bar{v} = \left(\frac{BR_m}{K_m} + K_e\right)\bar{\omega} - \left(\frac{R_m}{K_m}\right)\hat{\tau}_L \tag{27}$$

$$\bar{\imath}_a = \frac{B}{K_m}\bar{\omega} - \frac{1}{K_m}\hat{\tau}_L. \tag{28}$$

It turns out that the inductor current i in the boost-converter cannot be differentially parameterized in the same manner as the armature circuit variables. This shortcoming is due to the fact that the output $y = \omega$ of the cascade system is of relative degree 3, i.e., in this case, the relative degree with respect to y is strictly lower than the system order, which is 4. It can be shown that this system is not differentially flat [6]. In other words, the system does not exhibit a single variable (called the flat output) that differentially parameterizes all system variables; the current i in particular. This parametrization can only be achieved in an indirect manner with the help of an additional variable of physical interest.

For this purpose, we propose an indirect parametrization of the variable i in terms of the stored energy at the converter. More precisely, we resort to the stored energy in the Boost-converter

$$H_B(t) = \frac{1}{2}L(i(t))^2 + \frac{1}{2}C(v(t))^2 \tag{29}$$

and plan its reference from an initial equilibrium point $t = t_{\text{ini}}$ to a final equilibrium point $t = t_{\text{fin}}$. In this approximative approach, the equilibrium points are connected by an interpolation with a Bézier type polynomial, here of tenth order, that is updated at any time when a change of the load torque estimate $\hat{\tau}_L$ is detected; see [8] and [9] for a first notion of online trajectory replanning. Therefore, the desired dynamically stored energy in the boost-converter may be expressed by

$$H_B^\star(t) = \begin{cases} \bar{H}_{B,\text{ini}}, & t < t_{\text{ini}} \\ \bar{H}_{B,\text{fin}}, & t > t_{\text{fin}} \\ \bar{H}_{B,\text{ini}} + \left(\bar{H}_{B,\text{fin}} - \bar{H}_{B,\text{ini}}\right) b\left(\frac{t - t_{\text{ini}}}{t_{\text{fin}} - t_{\text{ini}}}\right), & \text{else} \end{cases} \tag{30}$$

with the 10th-order Bézier polynomial

$$b(t) = 252\,t^5 - 1050\,t^6 + 1800\,t^7 - 1575\,t^8 + 700\,t^9 - 126\,t^{10}. \tag{31}$$

The equilibrium energies follow from (1) and (2). Hence

$$\bar{H}_{B,\text{ini}} = \frac{1}{2}L\bar{\imath}_{\text{ini}}^2 + \frac{1}{2}C\bar{v}_{\text{ini}}^2 = \frac{1}{2}L\left(\frac{R_L^{-1}\bar{v}_{\text{ini}}^2 + \bar{\imath}_{a,\text{ini}}\bar{v}_{\text{ini}}}{E}\right)^2 + \frac{1}{2}C\bar{v}_{\text{ini}}^2 \tag{32}$$

and

$$\bar{H}_{B,\text{fin}} = \frac{1}{2}L\bar{\imath}_{\text{fin}}^2 + \frac{1}{2}C\bar{v}_{\text{fin}}^2 = \frac{1}{2}L\left(\frac{R_L^{-1}\bar{v}_{\text{fin}}^2 + \bar{\imath}_{a,\text{fin}}\bar{v}_{\text{fin}}}{E}\right)^2 + \frac{1}{2}C\bar{v}_{\text{fin}}^2 \tag{33}$$

respectively, and may be derived from the initial and final values of the stationary armature circuit variables, see (27) and (28), as per

$$\bar{v}_{\text{ini}} = \left(\frac{BR_m}{K_m} + K_e\right)\bar{\omega}_{\text{ini}} - \frac{R_m}{K_m}\hat{\tau}_L \tag{34}$$

$$\bar{\imath}_{a,\text{ini}} = \frac{B}{K_m}\bar{\omega}_{\text{ini}} - \frac{1}{K_m}\hat{\tau}_L \tag{35}$$

$$\bar{v}_{\text{fin}} = \left(\frac{BR_m}{K_m} + K_e\right)\bar{\omega}_{\text{fin}} - \frac{R_m}{K_m}\hat{\tau}_L \tag{36}$$

$$\bar{\imath}_{a,\text{fin}} = \frac{B}{K_m}\bar{\omega}_{\text{fin}} - \frac{1}{K_m}\hat{\tau}_L. \tag{37}$$

Finally, from $H_B^\star(t)$ one may easily compute the respective nominal trajectories for the inductor current and the control input from

$$i^\star(t) = \sqrt{\frac{1}{L}\left(2\,H_B^\star(t) - C(v^\star(t))^2\right)} \tag{38}$$

$$u_{\text{av}}^\star(t) = \frac{1}{v^\star(t)}\left(E - L\frac{di^\star(t)}{dt}\right) = \frac{1}{v^\star(t)}\left(E - \left(\frac{\dot{H}_B^\star(t) - Cv^\star(t)\dot{v}^\star(t)}{i^\star(t)}\right)\right). \tag{39}$$

Note that by an appropriate design of the circuit parameters and by the choice of the nominal angular velocity profile $\omega^\star(t)$ the positivity of the argument of the square root in (38) can be guaranteed in practice.

V. Algebraic Online Estimation of the Load Torque

This section applies the algebraic approach to model-based, online parameter estimation, to the problem of online load estimation in our boost-converter/dc-motor system (see [5]). The essential difference of the algebraic approach to traditional parameter estimation techniques lies in the fact that a valid, static, formula for the parameter determination is sought, derived on the basis of an assumed correct physical model, which is instantaneously usable, or evaluated, in terms of actual, on going, measured signals. This is in contradistinction to the introduction of an entire dynamic system which eventually produces the parameter estimate in an asymptotic fashion where the effect on the closed-loop stability has to be assessed. In the algebraic approach, once the parameter formula is evaluated, or computed (usually in a very small amount of time), its numerical result is replaced, wherever needed, in the controller expression. The long-term effects of the short lived errors occurring during the parameter formula evaluation or computational resetting in the interval are here assumed to be taken care of by the controller, as if, equivalently, perturbations in the feedback loop had occurred during this small time interval.

Our basic assumption is that the uncertain load is piecewise constant. The local estimates of the load values are achieved in a rather short time interval by means of a, model based, instantaneous estimation formula devoid of asymptotic convergence features. The feedback signal errors are then ascribed to the feedforward term where the load value is required and, then, only to those small time intervals where the updating load calculations are being carried out, or are being reset. As advocated below and shown in the experimental results, these time intervals, of length δ, are indeed insignificantly small. Although the rather small effects of such temporary feedback signal errors in the overall closed-loop behavior are not rigorously tackled here, it is clear that the passivity based feedback part of the control strategy is robust enough to compensate for such temporary mismatches in the feedforward signal during the small, load calculation resetting intervals.

The calculation of the reference trajectories is based on an estimate of the unknown load torque τ_L. In order to avoid the detrimental influence of high-level measurement noise on the armature current, and so as to render the estimation possible with linear techniques from operational calculus, we propose an energy-based algebraic approach to load torque estimation. To this end, we denote the total stored energy of the system in the form

$$H(x) = \frac{1}{2}\underbrace{\left(Li^2 + Cv^2 + L_m i_a^2 + J\omega^2\right)}_{=:z} \tag{40}$$

and use the system (1)–(4) in order to determine

$$\begin{aligned}\frac{dH}{dt} &= \frac{1}{2}\dot{z} = L\frac{di}{dt}i + C\frac{dv}{dt}v + L_m\frac{di_a}{dt}i_a + J\frac{d\omega}{dt}\omega \\ &= -\underbrace{\left(\frac{1}{R_L}v^2 + R_m i_a^2 + B\omega^2 - iE\right)}_{=:y} + \tau_L\,\omega = -y + \tau_L\,\omega.\end{aligned} \tag{41}$$

From the lines above it is easy to see that

$$\tau_L\,\omega = \frac{1}{2}\dot{z} + y. \tag{42}$$

We assume that the load torque τ_L be piecewise constant and derive the operational calculus version of (42). We obtain

$$\tau_L\,\Omega(s) = \frac{1}{2}\big(s\,Z(s) - z(0)\big) + Y(s). \tag{43}$$

In order to get rid of the initial value $z(0)$ we may differentiate (43) with respect to the operator s. Therefore

$$\tau_L\,\frac{d}{ds}\Omega(s) = \frac{1}{2}\left(Z(s) + s\,\frac{d}{ds}Z(s)\right) + \frac{d}{ds}Y(s). \tag{44}$$

Multiplying (44) by $1/s$ we eliminate time-derivatives, thus

$$\tau_L\left(\frac{1}{s}\frac{d}{ds}\Omega(s)\right) = \frac{1}{2}\left(\frac{1}{s}Z(s) + \frac{d}{ds}Z(s)\right) + \frac{1}{s}\frac{d}{ds}Y(s). \tag{45}$$

For finding τ_L in the time-domain the inverse transform is applied to (45). This yields

$$\tau_L = \frac{\frac{1}{2}\left(\int_{t_i}^{t} z(\sigma)\,d\sigma - (t-t_i)\,z(t)\right) - \int_{t_i}^{t}(\sigma - t_i)y(\sigma)\,d\sigma}{-\int_{t_i}^{t}(\sigma - t_i)\,\omega(\sigma)\,d\sigma}. \tag{46}$$

This equation indicates that an estimate $\hat{\tau}_L$ of the unknown load torque may be expressed in terms of the measurable states i, v, i_a, and ω. In light of this, we propose to use the estimate

$$\hat{\tau}_L = \begin{cases} \hat{\tau}_L(t_i^-) & \text{for } t \in [t_i, t_i + \delta] \\ \frac{n(t)}{d(t)} & \text{for } t > t_i + \delta \\ \text{with} & \\ n(t) = & 0.5\int_{t_i}^{t}(Li^2 + Cv^2 + L_m i_a^2 + J\omega^2)\,d\sigma \\ & -0.5(t-t_i)(Li^2 + Cv^2 + L_m i_a^2 + J\omega^2) \\ & -\int_{t_i}^{t}(\sigma - t_i)\left(\frac{1}{R_L}v^2 + R_m i_a^2 + B\omega^2 - iE\right)d\sigma \\ d(t) = & -\int_{t_i}^{t}(\sigma - t_i)\,\omega(\sigma)\,d\sigma \\ \text{and} & \\ t_i = & kT, k = 0, 1, 2, \ldots, \quad T \gg \delta. \end{cases} \tag{47}$$

Formula (47) comprises periodic resettings at the end of time intervals of duration T. These resettings take into account that though the load torque is assumed piecewise constant it may still vary in time. Hence by resetting, past measurement values are discarded in the estimator in favor of more recent measurement values containing up-to-date information of the actual load torque. For those reasons, the duration T needs to be adapted to the expected time scale of the load changes, which may require some judgement to be drawn from experiments. The duration δ in formula (47) is to be considered a strict lower bound for the period T and is a simple means for tackling the problem of indetermination when dividing small values by small values under finite precision. The value of δ needs to be chosen large enough also because noise is affecting the measurement signals, which then would cause the same indetermination. In the experiments we found that it was sufficient to keep the past estimated value $\hat{\tau}_L(t_i^-)$ as an estimate of torque for the duration of δ, that is, a small period of rest was implemented. Choosing the periodic resetting duration T by one magnitude larger than the period of rest δ, turned out to yield acceptable results for the estimate.

For other systems like a buck-boost converter cascaded with a dc-motor [14], the estimation of the load torque may be carried out along the same lines, but with the slight difference that the variable y in (42) will then also contain the average control.

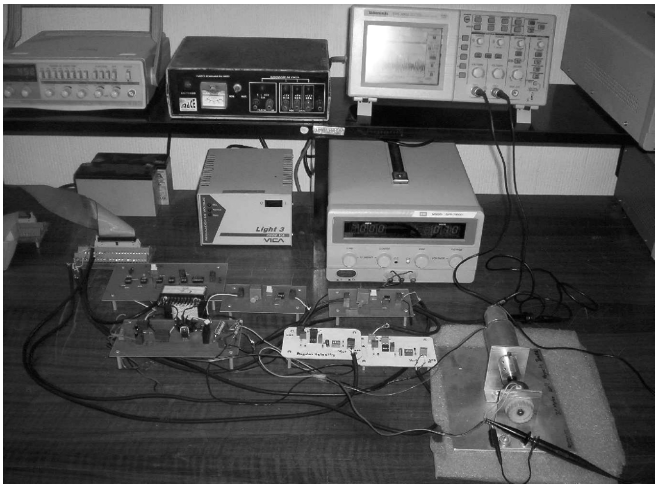

Fig. 2. Experimental hardware setup.

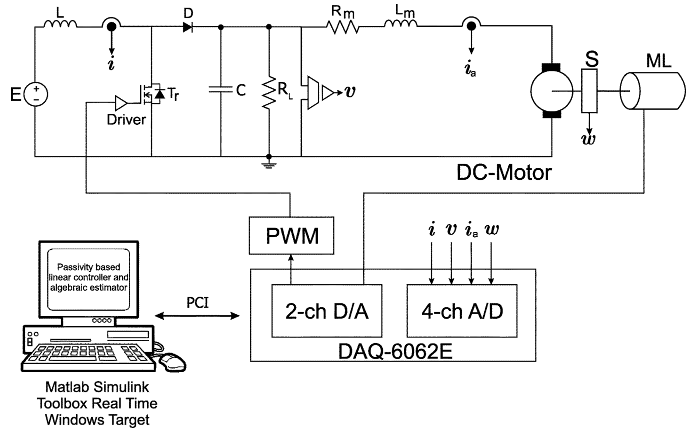

Fig. 4. Schematic of the experimental setup of the boost-converter/dc-motor system and DAQ-6062E data-acquisition card in closed-loop.

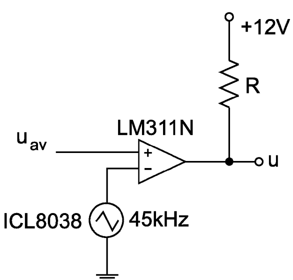

Fig. 3. PWM-circuit: triangular generator (ICL8038) and comparator circuit (LM311N).

VI. Experimental Setup for the Boost-Converter/DC-Motor System

The experimental hardware setup depicted in Fig. 2 is composed of the following devices: a permanent magnet dc-motor of type Dunkermotoren GR 42×25 with tacho generator TG-11, two current sensor circuits, one for the input current of the power converter and an other for the armature current of the dc-motor, a voltage sensor circuit for the armature voltage, an external pulse width modulator circuit (45 kHz), a data acquisition card that serves as a link between the analog circuits and the computer through the Real-Time Windows Target Simulink Library, a computer with MATLAB/Simulink to implement the average controller, and the mechanical-electrical interface that activates/disables the load torque in the motor shaft. The respective schematic is given in Fig. 4.

The data-acquisition card of type DAQ-6062E was used as a hardware interface to MATLAB/Simulink where the algorithm of the linear time-varying feedback controller and the algebraic estimator of the system's load torque was implemented. An external PWM-modulator (for the circuit see Fig. 3) was employed to command the switch position function of the converter. Conventionally, the duty ratio of the PWM is set to the average continuous linear, time-varying feedback control law. The PWM-modulator was implemented externally in order not to reduce the transfer capacity of the data acquisition card which can be driven faster than 45 kHz. The mechanical load is controlled through an external pulse imposed by a channel of the acquisition card (see Fig. 4).

The design parameters chosen for the boost-converter are: $E = 7$ V, $C = 57.6\,\mu$F, $L = 15.91$ mH, $R_L = 492.6\ \Omega$. The motor parameters $K_e = K_m$ (back emf and mechanical power, respectively) and the viscous friction coefficient B were estimated statically through the electrical and mechanical equations (3) and (4), whereas the values of the parameters L_m and J were taken from the time responses of current and speed in open loop [11]. The armature resistance was determined from Ohm's law from current and voltage measurements when feeding the motor with a nominal voltage and blocking its motor shaft [19]. All values have been compared and validated with the data sheet values of the motor manufacturer (Dunkermotoren).

The sampling time of the program controller was set to 220 μs. We have specified a controller gain of $\gamma = 0.150$, rendering $\tilde{R}$ positive definite. The specific value of γ helps reduce the impact of the measurement noise on the feedback so as to keep the average control signal within its bounds $[0, 1]$ (see middle right plot in Fig. 6) while guaranteeing an acceptable tracking of the angular velocity. The specifications of the boost-converter/dc-motor system and of the algebraic estimator for the system's load torque may be found in Table I.

VII. Experimental Results

A nominal desired angular velocity profile that exhibits a rather smooth start for the dc-motor was specified using an interpolating Bézier polynomial of 10th-order, as in (30), hence

$$\omega^\star(t) = \begin{cases} \bar{\omega}_{\text{ini}}, & t < t_{\text{ini}} \\ \bar{\omega}_{\text{fin}}, & t > t_{\text{fin}} \\ \bar{\omega}_{\text{ini}} + (\bar{\omega}_{\text{fin}} - \bar{\omega}_{\text{ini}})\, b\left(\frac{t - t_{\text{ini}}}{t_{\text{fin}} - t_{\text{ini}}}\right), & \text{else} \end{cases} \tag{48}$$

with the 10th-order Bézier polynomial as given in (31). Here, the initial angular velocity was set to be $\bar{\omega}_{\text{ini}}(1.5\ \text{s}) = 200$ rad/s and the final desired value of the angular velocity was specified as $\bar{\omega}_{\text{fin}}(2.2\ \text{s}) = 300$ rad/s. The resulting input current, armature current, output voltage, and average control are depicted in

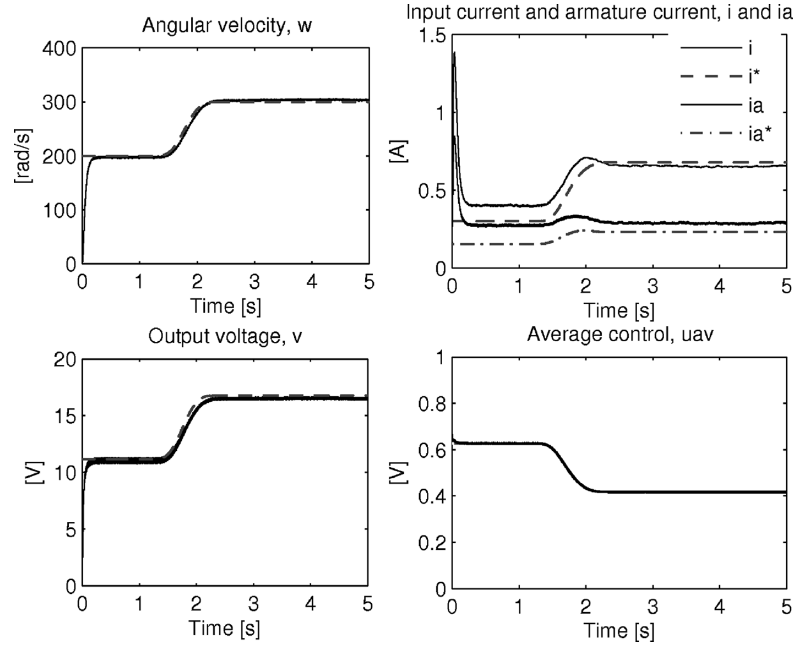

Fig. 5. Experimental results: angular velocity, input and armature current, converter output voltage, and average control input for a desired angular velocity trajectory tracking task with no external load torque, $\tau_L = 0$—reference signals (dashed or dashed-dotted lines), measured signals (solid lines).

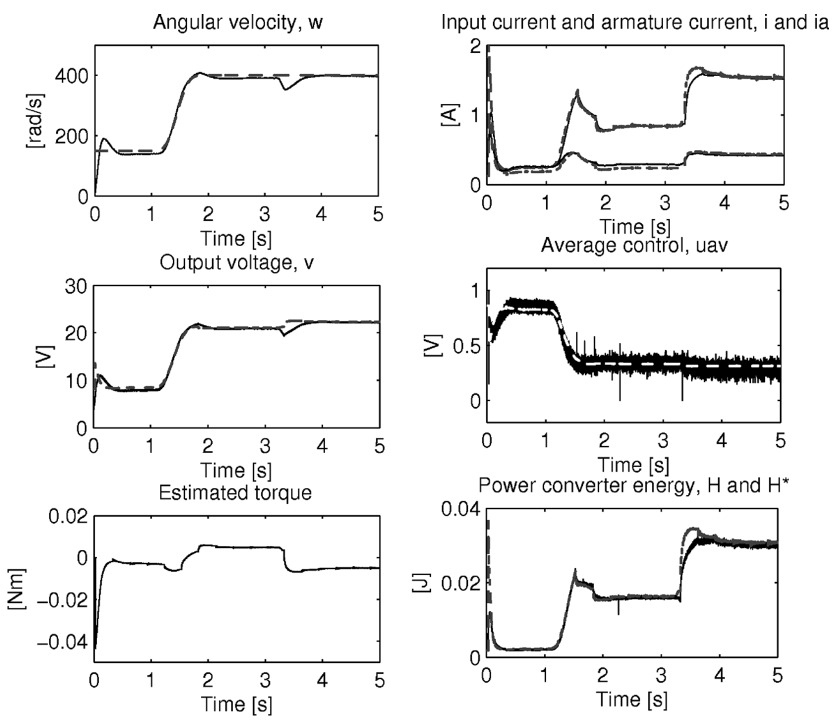

Fig. 6. Experimental results: angular velocity, input and armature current, converter output voltage, average control input, estimated load torque, power converter energy for a desired angular velocity trajectory tracking task with external load torque, $\tau_L \neq 0$—reference signals (dashed or dashed-dotted lines), measured signals (solid lines).

Fig. 5. The average control signal remains within the bounds of $[0, 1]$.

Fig. 6 illustrates a transition from $\bar{\omega}_{\text{ini}}(1\ \text{s}) = 150$ rad/s to $\bar{\omega}_{\text{fin}}(2\ \text{s}) = 400$ rad/s, but in the case with varying load torque. The deviation between the input current and armature current and their respective nominal values are significantly reduced. This behavior resembles the online accommodation of the corresponding nominal trajectories to the actual load torque estimate. Despite of the rapid load torque changes imposed on the DC-drive the angular velocity tracking performance is quite satisfactory, while the average control signal remains within its bounds. Note the slight riple on the references after every period of $T = 0.3$ s. On the measured signals the ripple is not visible anymore.

TABLE I
SPECIFICATIONS OF THE BOOST-CONVERTER/DC-MOTOR SYSTEM AND THE LOAD TORQUE ESTIMATOR

System Parameters	
Power rating	11W
Switching device	IRF640 (200 V/18A)
Commutation frequency	45kHz
Acquisition device	DAQCard-6062E
Power diode	MBRF20100CT
Control method	Passivity-based linear controller
Supply voltage	$E = 7\,\mathrm{V}$
DC capacitor	$C = 57.6\,\mu\mathrm{F}$
Boost inductor	$L = 15.91\,\mathrm{mH}$
Resistor load	$R_\mathrm{L} = 492.6\,\Omega$
DC-Motor	GR42x25 Dunkermotoren
Armature resistance	$R_\mathrm{m} = 6.14\,\Omega$
Armature inductance	$L_\mathrm{m} = 8.9\,\mathrm{mH}$
Viscous friction coefficient	$B = 40.92\,\mu$(Nm-s)/rad
Moment of intertia (motor & tacho)	$J = 7.95\,\mu\mathrm{kg}m^2$
Electrical constant	$K_\mathrm{e} = 0.04913\,\mathrm{Vs/rad}$
Torque constant	$K_\mathrm{m} = 0.04913\,\mathrm{N\text{-}m/A}$
Sampling time	$220\,\mu\,\mathrm{sec}$

Estimator Parameters	
Period of rest	$\delta = 0.03$ sec
Periodic reset time	$T = 0.3$ sec

VIII. CONCLUSION

In this brief, a control for a "smooth starter" is developed for the angular velocity regulation along a desired angular shaft velocity profile of a boost-converter driven dc-motor. Since the overall model of the boost-converter/dc-motor combination shows unstable internal dynamics (input current) with respect to the angular shaft velocity as the output, a passivity-based controller is devised. The result is a time-varying feedback controller that, together with a feedforward precompensator, stabilizes a valid reference trajectory of the system. The reference trajectory required by the controller is generated in an approximate manner, resorting to the stored energy in the boost-converter. For robustness, the reference trajectories receive an update for the nominal load torque parameter whenever the online-generated estimate of the load torque changes. To this end, a simple so-called algebraic parameter estimator is designed which is subject to periodic resettings, for maintaining accuracy. Despite of the simplicity of the approach, however, the experimental results are promising. This gives reason to expect a successful application of this concept on other topologies.

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