Discrete Event-Triggered Sliding Mode Control With Fast Output Sampling Feedback

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***Abstract*—This paper deals with event-triggering based design of discrete-time sliding mode (DTSM) control for linear systems. Multirate output feedback based DTSM is designed with event-triggering strategy such that the system trajectories remain bounded in the vicinity of sliding manifold. An event-triggering rule is developed for DTSM which is evaluated only at periodic time intervals. The control is updated whenever the triggering instant is generated at these discrete time instants. It is shown that with this technique the system trajectories remain bounded. Also, in this triggering scheme there is always guaranteed *Zeno* free execution of triggering sequences as it is inherently discrete. Simulation results are shown to demonstrate the theoretical treatments of this paper.**

***Index Terms*—Event-triggered control, discrete-time sliding mode control, multirate output feedback.**

I. Introduction

Sliding mode control (SMC) is a robust control methodology which ensures complete disturbance rejection (for a class of external disturbances) and achieves robust stabilization of the system [1]– [3]. The disturbance rejection occurs once the sliding mode is enforced in the system. As a result of the sliding mode, the system trajectory slides along a predesigned stable manifold which ensures stability of sliding mode dynamics. SMC has been successfully applied in many practical applications with external uncertainties. Many developments in this area may be found in [3].

In real time applications, the control law is implemented digitally due to the availability of reliable and sophisticated digital platforms. So, the discrete counterpart of SMC is also investigated by many researchers [4]– [14]. It is observed that in discrete-time sliding mode (DTSM), the system trajectory reaches the sliding manifold but does not remain on this. So, many research have been published on stability and convergence of DTSM and few of these are listed here [8], [9], [4], etc. In all these cases the control is implemented at discrete instants periodically. However, very few papers have discussed aperiodic implementation of DTSM control laws. Though the aperiodic implementation is always desired in many application, the stability of the system may not be ensured. Event-triggered control implementation is one such strategy where control is implemented aperiodically while guaranteed system stability.

The main idea of this paper is to design DTSM control with event-triggering technique. It is an implementation strategy where control is applied only when a certain triggering rule is violated [15]– [17], [23], [29], [30]. This rule necessitates that the deviation of system trajectory from its immediate previous sampled value crosses the certain threshold value for triggering to be generated. Thus, the control is applied if there is a demand making the closed loop system more energy efficient. This technique has many advantages such as in networked control system where the larger time interval between two control update is desired for less communication among control end and plant [18], [19].

Event-triggering strategy with respect to external disturbances is not studied much. So, event-triggered SMC is first proposed in [24]– [28] and is shown that the sliding trajectories are confined within a predesigned band. The size of this band can be varied by suitably selecting the design parameters. However, in event-triggering the state trajectory is continuously measured to generate the possible triggering instant. So, this needs a dedicated circuit/sensor to realize this strategy, which may increase the cost of the practical system. In order to overcome this difficulty, another approach, motivated from event-triggering, is proposed called self-triggering. Unlike the vent-triggering, here no extra circuit is required for realizing this strategy. The next triggering instant is calculated from the past sampled value only, thus avoiding no dedicated circuit. The recent developments in self-triggering may be found in [20]– [23]. Motivated from this strategy, the self-triggered SMC is also proposed in [25].

In this paper, we propose a event-triggering strategy where event is checked only at discrete instants. The control is updated whenever the event is violated at discrete instants. Since the triggering instant is generated at discrete instants only, there is no Zeno execution occurs in this realization. The DTSM control discussed in the paper uses Bartoszewicz's reaching law to show the convergence to discrete sliding manifold [6]. The control law is designed using this reaching law and is applied to the plant whenever an event has occurred. In the next, output feedback strategy is used to design the DTSM control which is proposed in [10]. The outputs are sampled between two discrete instants and state is computed from these output measurements. Then, triggering instant is generated if event is violated.

The paper is organised as follows. Section II introduces the DTSM control design using Bartoszewicz's reaching law and also the stability of the system. In Section III, event-triggered SMC with DTSM control is proposed. Also, event-triggering rule is stated in this Section. Then, multirate output feedback (MROF) using fast output sampling (FOS) is given in Section IV. The theoretical results are demonstrated through the simulation results in Section V. Finally, some concluding remarks are given in Section VI.

II. Discrete-Time Sliding Mode

In this Section, we discuss DTSM control for the discrete-time system briefly. Consider a single-input single-output discrete-time dynamical system as

$$x(k+1) = Ax(k) + B(u(k) + d(k)) \tag{1}$$

with $x_0 = x(0)$ and $x \in \mathbb{R}^n$. Here, we deal with only a class of disturbances which enters the system through input channel only. It is further assumed that the disturbance d is bounded, i.e., $|d(k)| \le d_0$ for all $k \in \mathbb{Z}_{\ge 0}$. We discuss the stabilization of the system (1) with DTSM control.

Design sliding variable $s = c^\top x$ where c is a column vector of appropriate dimension. The sliding manifold is defined as

$$\mathcal{S} \stackrel{\text{def}}{=} \left\{ x \in \mathbb{R}^n : s = c^\top x = 0 \right\}. \tag{2}$$

It is well known that in DTSM the sliding variable $s(k)$ does not go to zero exactly at each discrete instants $k \in \mathbb{Z}_{\ge 0}$ in the presence of disturbance due to inherent nature discrete structure of control algorithm. As a result, the system trajectory remain does not slide on the manifold $\mathcal{S}$, but remains bounded in the vicinity of this manifold. This is often known as quasi sliding mode band (QSMB) in literature.

Definition 2.1 (Quasi Sliding Mode): Consider the sliding variable $s = c^\top x$. The system (1) is said to be in quasi sliding mode if given any $\epsilon > 0$ and $\bar{k} \ge 0$ the trajectory of the system evolves such that $s(k) = c^\top x(k)$ satisfies

$$|s(k)| \le \epsilon$$

for all $k \ge \bar{k}$. The constant $\epsilon > 0$ is the size of QSMB.

It has been studied by many researchers to bring the sliding variable $s(k)$ to zero in successive time steps. However, in this paper we study Bartoszewicz's reaching law for stability of DTSM control. Compared other reaching laws, this law does not necessitates the crossing and recrossing of s about sliding manifold $\mathcal{S}$ in every successive discrete steps.

A. Bartoszewicz's Reaching Law

Consider the reaching law proposed by Bartoszewicz [6]

$$s(k+1) = \tilde{d}(k) + s_d(k+1) \tag{3}$$

where $\tilde{d}(k)$ is an uncertain variable to de defined later with $\sup_{k \in \mathbb{Z}_{\ge 0}} |\tilde{d}(k)| \le \tilde{d}_0$ and $s_d(k)$ is *a priori* known function such that followings hold:

- If $|s(0)| > 2\tilde{d}_0$, then

$$\begin{aligned} s_d(0) &= s(0) \\ s_d(k) \cdot s(0) &\ge 0, \quad \text{for any } k \ge 0 \\ s_d(k) &= 0, \quad \text{for any } k \ge k^\star \\ |s_d(k+1)| &< |s_d(k)| - 2\tilde{d}_0, \quad \text{for any } k < k^\star. \end{aligned}$$

- Otherwise, i.e., if $|s(0)| \le 2\tilde{d}_0$, then $s_d(k) = 0$ for any $k \ge 0$.

The above reaching law ensures for $|s(0)| > 2\tilde{d}_0$, the sliding variable $s(k)$ decreases in each discrete step by an amount more than $2\tilde{d}_0$. The constant $k^\star$ is a positive integer, and it confines the rate of convergence of $s(k)$ to QSMB in $k^\star$ steps. The value of $k^\star$ can be chosen to meet the design constraint and rate of convergence to the manifold $\mathcal{S}$. One such choice of $s_d(k)$ is given as

$$s_d(k) = \frac{k^\star - k}{k^\star} s(0), \qquad k^\star < \frac{|s(0)|}{2\tilde{d}_0}.$$

To design DTSM control for the system (1), we write

$$\begin{aligned} s(k+1) &= c^\top x(k+1) \\ &= c^\top Ax(k) + c^\top Bu(k) + c^\top Bd(k). \end{aligned} \tag{4}$$

Denote $\tilde{d} = c^\top Bd$ for simplicity. The DTSM control for the system dynamics (4) is given as

$$u(k) = -\left(c^\top B\right)^{-1} \left(c^\top Ax(k) - s_d(k+1)\right). \tag{5}$$

The closed loop system can now be written as

$$s(k+1) = \tilde{d}(k) + s_d(k+1). \tag{6}$$

We see that for all $k \ge k^\star$, the sliding dynamics (6) is reduced to $s(k+1) = \tilde{d}(k)$ and the trajectory remain bounded with an ultimate bound given by

$$|s(k)| = |\tilde{d}(k)| \le \tilde{d}_0. \tag{7}$$

So, in DTSM control the trajectory remains bounded with the control applied at every periodic steps. Here, in this paper we mainly emphasize the stabilization of the discrete time system with minimum control updates. This means the control may not be applied at every periodic instants but whenever it is demanded. One of such technique which realizes the control with minimum control updates is the event-triggering strategy. So, the main objective in the following Section is to devote the attention to discrete-time event-triggered SMC.

III. Event-Triggered Sliding Mode Control

In discrete-time system the control is applied at every periodic intervals of time. It may be noted that the control is not necessarily needed at every periodic instant always to ensure the stability of closed loop system. Thus, there has been numerous interest in research to update the control at aperiodic instants, and subsequently achieving maximum inter control execution (update) time interval. One of such techniques is the event-triggering strategy which stabilizes a system with minimum control updates.

Definition 3.1 (Practical Quasi Sliding Mode): Let $x(k, x_0)$ be the system trajectory starting from initial condition $x_0 = x(0)$. The system is said to be in practical quasi sliding mode if given $\epsilon_1 > 0$ and $\bar{k} > 0$ the system trajectory evolves such that $s(k) = c^\top x(k)$ ensures the system is in quasi sliding mode, and satisfies

$$|s(k)| \le \epsilon_1$$

for all $k \ge \bar{k}$. The constant $\epsilon_1 > 0$ is the size of practical QSMB.

Remark 3.1: From the above Definitions, it is clear that the band size of practical QSMB is always greater than or equals than that of QSMB.

We write $\{k\tau\}_{k\in\mathbb{Z}_{\ge 0}} = \{k\}_{k\in\mathbb{Z}_{\ge 0}}$ for a given discrete time interval $\tau > 0$. Let $\{k_i\}_{i\in\mathbb{Z}_{\ge 0}}$ be the sequence of control update instants. If the DTSM control law (5) is implemented at every $\{k_i\}_{i\in\mathbb{Z}_{\ge 0}}$ rather than at periodic time intervals, then we can write it as

$$u(k) = -\left(c^\top B\right)^{-1}\left(c^\top Ax(k_i) - s_d(k_i+1)\right) \qquad (8)$$

for all $k \in [k_i, k_{i+1})$. From this, we note that the next control update instant k_{i+1} does not necessarily equal $k+1$. We define $e(k) = x(k_i) - x(k)$ as the error introduced in the system due to implementation of control (8) aperiodically. Note that in the control (8), known function $s_d(k)$ takes value at $k = k_i + 1$, not at k_{i+1}. This is to ensure that the convergence to practical QSMB can be achieved at desired $k^\star$ steps. And, also the time instant k_{i+1} is not known apriori.

In this paper, we discuss the event-triggered realization of DTSM control using Bartoszewicz's reaching law. Since practical QSMB is more than the QSMB, the reaching law given Bartoszewicz is not applicable in event-triggered SMC. This is because to ensure $s_d(k) = 0$ once it reaches the practical QSMB. So in the below we provide the modified Bartoszewicz's reaching law.

A. Event-Triggered Bartoszewicz's Reaching Law

Bartoszewicz's reaching law is modified in the context of discrete event-triggered SMC to achieve practical QSMB and is given as

$$s(k+1) = \tilde{d}(k) + \alpha + s_d(k+1) \qquad (9)$$

where $\tilde{d}(k)$ and $s_d(k)$ are defined as earlier, and $\alpha > 0$. Then the followings hold:

- If $|s(0)| > 2(\tilde{d}_0 + \alpha)$, then

$$\begin{aligned} s_d(0) &= s(0) \\ s_d(k)\cdot s(0) &\ge 0, \quad \text{for any } k \ge 0 \\ s_d(k) &= 0, \quad \text{for any } k \ge k^\star \\ |s_d(k+1)| &< |s_d(k)| - 2(\tilde{d}_0 + \alpha), \quad \text{for any } k < k^\star. \end{aligned}$$

- Otherwise, i.e., if $|s(0)| \le 2(\tilde{d}_0 + \alpha)$, then $s_d(k) = 0$ for any $k \ge 0$.

This modified Bartoszewicz's reaching law is different from that proposed by Bartoszewicz. This ensures in each time step the sliding variable decreases by a magnitude greater than $2(\tilde{d}_0 + \alpha)$. The function $s_d(k)$ in the case of event-triggered DTSM can be given as

$$s_d(k) = \frac{k^\star - k}{k^\star} s(0), \qquad k^\star < \frac{|s(0)|}{2(\tilde{d}_0 + \alpha)}.$$

The following Theorem states the main result of the paper.

Theorem 3.1: Consider the system (1) and the control law (8). Let $\{k_i\}_{i\in\mathbb{Z}_{\ge 0}}$ be the sequence of control update instants. Let $\alpha > 0$ be given. Then the system is in practical quasi sliding mode if

$$\left|c^\top Ae(k)\right| < \alpha \qquad (10)$$

for all $k \in \mathbb{Z}_{\ge 0}$.

Proof: Consider the case $k \in [k_i, k_{i+1})$ for all $i \in \mathbb{Z}_{\ge 0}$ such that $k_i < k^\star$. Using DTSM control (8) we can write

$$\begin{aligned} s(k+1) &= c^\top x(k+1) \\ &= c^\top Ax(k) + c^\top Bu(k) + c^\top Bd(k) \\ &= -c^\top Ae(k) + \tilde{d}(k) + s_d(k_i+1). \end{aligned} \qquad (11)$$

Using (10) and $|\tilde{d}(k)| \le \tilde{d}_0$ in (11), yields

$$\begin{aligned} s(k+1) &\le |c^\top Ae(k)| + |\tilde{d}(k)| + s_d(k_i+1) \\ &< \alpha + \tilde{d}_0 + s_d(k_i+1). \end{aligned} \qquad (12)$$

So, it leads to

$$|s(k+1)| < \alpha + \tilde{d}_0 + |s_d(k_i+1)|. \qquad (13)$$

To see how $s(k)$ converges to the practical QSMB, we consider the evolution of both $s_d(k)$ and $s(k)$. Since $s_d(k)$ decreases by a magnitude at least equal to $2(\tilde{d}_0 + \alpha)$ in every discrete time step. So, between two consecutive triggering instants, it decreases by $2(k_{i+1} - k_i)(\tilde{d}_0 + \alpha)$. As a result of this and due to (13), the sliding variable $s(k)$ also decreases by an amount at least equal to $(k_{i+1} - k_i)(\tilde{d}_0 + \alpha)$ during the time interval $[k_i, k_{i+1})$.

Once k_i reaches $k^\star$-th step, then for all $k^\star \le k \in [k_i, k_{i+1})$ we have $s_d(k) = 0$. So, the above relation (13) further reduces to

$$|s(k)| < \alpha + \tilde{d}_0, \qquad \forall k \ge k^\star.$$

This proves that the system is in practical quasi sliding mode. The practical QSMB can be given as

$$\Omega = \left\{x \in \mathbb{R}^n : |s| = \left|c^\top x\right| < \epsilon_1\right\} \qquad (14)$$

where $\epsilon_1 := \alpha + \tilde{d}_0$. ■

B. Event-Triggering Rule

Here, we explicitly discuss the event-triggering rule for DTSM control. It is seen from Theorem 3.1 that the stability of the system is established provided the relation (10) holds for all $k \in \mathbb{Z}_{\ge 0}$. Hence, this must be ensured for all time to guarantee system stability. This may be chosen to generate the triggering rule for the system (1). However, a more stronger condition than (10) is realized for event-triggering such that

$|c^\top Ae| < \sigma\alpha$ is satisfied for some $\sigma \in (0,1)$ in order to compensate for unavoidable delays/computations. This is expressed as

$$k_{i+1} = \inf\{k > k_i : \|c\|\|A\|\|e(k)\| \geq \sigma\alpha\}. \quad (15)$$

The triggering rule (15) generates a sequences of triggering instants $\{k_i\}_{i=1}^{\infty}$ whenever the strengthened relation (10) is violated. Thus, the results of the above Theorem holds. It is to be noted that this discrete triggering scheme guarantees no Zeno execution of triggering sequences due to inherent discrete structure.

Now, we discuss the proposed event-triggering scheme with multirate output feedback in the following Section.

IV. Design With Multirate Output Feedback

In the following, we discuss the design of event-triggered SMC with multirate output feedback. Fast output sampling (FOS) is used to realize the multirate output feedback technique. In FOS the outputs are sampled at a faster rate than the input applied to the plant. The control input to the system feeds back the outputs sampled over a time interval of input sampling time. This technique is called FOS.

Let the output of discrete-time dynamical system is sampled at every Δ interval and input is applied to the plant at every τ interval. We consider the system (1) is given which evolves at τ interval. We let the system sampled at every Δ interval given as

$$x(k+1) = A_\Delta x(k) + B_\Delta u(k) + B_\Delta d(k) \quad (16)$$

$$y(k) = Cx(k). \quad (17)$$

Here, Δ is chosen as $\Delta = \tau/N$ where N is greater than or equal to observability index of the system (16)–(17). Note that the disturbance $d(k)$ does not change at Δ instants since the closed loop system evolves at τ interval. Outputs are sampled at every Δ interval and these information of output are applied to the plant at τ intervals. The advantage of this technique is that with this stack of output the closed-loop eigenvalues of the system can be placed at any desired place. The detailed analysis is presented in [10], [11].

It can be easily seen that from Δ system (16) we arrive at

$$x((k+1)\tau) = A_\Delta^N x(k\tau) + \sum_{i=0}^{N-1} A_\Delta^i B_\Delta u(k\tau) + \sum_{i=0}^{N-1} A_\Delta^i B_\Delta d(k\tau).$$

From analogy with τ system (1), we obtain

$$A = A_\Delta^N \quad \text{and} \quad B = \sum_{i=0}^{N-1} A_\Delta^i B_\Delta.$$

Now combine all these together and write it in compact form, it results

$$x(k+1) = Ax(k) + Bu(k) + Bd(k) \quad (18)$$

$$y_{k+1} = C_0 x(k) + D_0 u(k) + D_0 d(k) \quad (19)$$

where

$$C_0 = \begin{bmatrix} C \\ CA_\Delta \\ \vdots \\ CA_\Delta^{N-1} \end{bmatrix}, \qquad D_0 = \begin{bmatrix} 0 \\ CB_\Delta \\ \vdots \\ C\sum_{i=0}^{N-2} A_\Delta^i B_\Delta \end{bmatrix},$$

and

$$y_k := \begin{bmatrix} y((k-1)\tau) \\ y((k-1)\tau + \Delta) \\ \vdots \\ y(k\tau - \Delta) \end{bmatrix}.$$

From (19), we write $x(k) = (C_0^\top C_0)^{-1} C_0^\top (y_{k+1} - D_0 u(k) - D_0 d(k))$. Substituting this in (18),

$$x(k+1) = L_y y_{k+1} + L_u u(k) + L_u d(k) \quad (20)$$

where

$$L_y = A(C_0^\top C_0)^{-1} C_0^\top$$
$$L_u = B - A(C_0^\top C_0)^{-1} C_0^\top D_0.$$

This gives the expression of the state from the fast output measurements, and past control input and disturbance. This relation may be used in place of state so that only output information appears in the control expression. It is to be noted that (20) contains disturbance, so in further analysis the following relation

$$\bar{x}(k+1) = L_y y_{k+1} + L_u u(k) \quad (21)$$

is used instead of (20).

The SMC can be designed using MROF as follows. The only difference in this case is the relation (21) is used wherever $x(k)$ appears. We use the reaching law proposed in [10, Section IV] and is written below as

$$s(k+1) = \tilde{d}(k) + f(k-1) + s_d(k+1) \quad (22)$$

where $f(k) = c^\top A L_u d(k)$ is the uncertain function with the known bound $\sup_{k\geq 0} |f(k)| \leq f_0$. The DTSM control with multirate output feedback can be given as

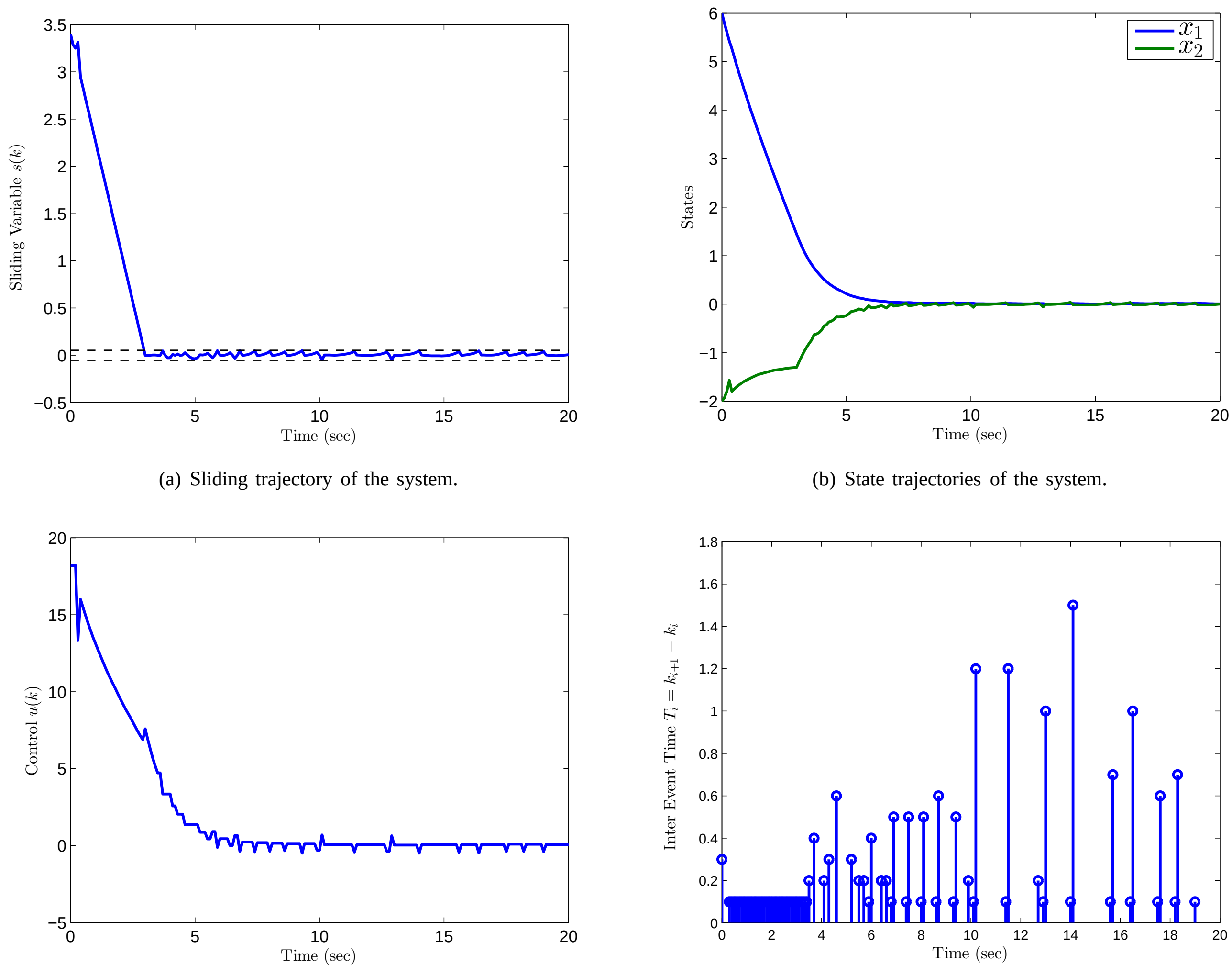
$$u(k) = -\left(c^\top B\right)^{-1} \left(c^\top A\bar{x}(k) - s_d(k+1)\right). \quad (23)$$

Using this control the closed loop sliding dynamics reduces to same as (22). However, the main objective of this Section is to analyse the event-triggering scheme with MROF SMC. We discuss how the FOS feedback based discrete event SMC is designed in the below.

A. Multirate Event-Triggered DTSM

Consider the control law (23). The FOS based feedback event-triggered DTSM control can be written using the relation (21) as

$$u(k) = -\left(c^\top B\right)^{-1} \left(c^\top A\bar{x}(k_i) - s_d(k_i+1)\right) \quad (24)$$



(a) Sliding trajectory of the system.

(b) State trajectories of the system.

(c) Sliding mode control with discrete event-triggered implementation.

(d) Inter event time between two consecutive control updates.

Fig. 1. Performance of discrete event-triggered sliding mode control with fast output sampling feedback.

for all $k \in [k_i, k_{i+1})$. Substituting the control (24) in the expression of sliding dynamics, it renders

$$\begin{aligned} s(k+1) = & -c^\top A\bar{e}(k) + \tilde{d}(k) + f(k-1) \\ & + s_d(k_i+1) \end{aligned} \tag{25}$$

where $\bar{e}(k) = \bar{x}(k_i) - \bar{x}(k)$. The stability of the above sliding dynamics (25) can be shown by invoking the event-triggered Bartoszewicz's reaching law with the new disturbance function $\tilde{f}(k) = \tilde{d}(k) + f(k-1)$. The minor modification required in the reaching law is the use of new disturbance bound given as $\sup_{k\geq 0} |\tilde{f}(k)| \leq \tilde{f}_0 := \tilde{d}_0 + f_0$. Thus, the relation (25) is rewritten as

$$s(k+1) = -c^\top A\bar{e}(k) + \tilde{f}(k) + s_d(k_i+1). \tag{26}$$

Another important difference between the sliding dynamics (26) and (11) is the discrete error $\bar{e}$ appearing in (26) not the e. The following Theorem states stability of (26).

Theorem 4.1: Consider the system (1) and the control law (24). Let $\{k_i\}_{i\in\mathbb{Z}_{\geq 0}}$ be the sequence of control update instants. Let $\alpha > 0$ be given. Then the system achieves practical quasi sliding mode if

$$\left|c^\top A\bar{e}(k)\right| < \alpha \tag{27}$$

for all $k \in \mathbb{Z}_{\geq 0}$.

Proof: Proof follows the similar lines as in Theorem 3.1 of course with some minor modifications. So, it is omitted here. ■

In the following, we discuss the event-triggering rule to achieve practical quasi sliding mode in the system with FOS feedback. It is concluded from Theorem that the relation (27) is essential to guarantee the desired practical QSMB for the system. So, triggering rule is to be developed which must ensure condition given by (27) always holds. This is given as

$$k_{i+1} = \inf\left\{k > k_i : \|c\|\|A\|\|\bar{e}(k)\| \geq \sigma\alpha\right\}.$$

V. Simulation Results

Consider a discrete-time dynamical system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u + d)$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x.$$

The values of τ and Δ are selected as 0.1 and 0.05 with $N = 2$, respectively. The disturbance d is assumed to be bounded by 0.01. The sliding surface is selected as $c = [0.9\ 1]^\top$ such that the reduced dynamics is asymptotically stable. It is desired that the sliding variable reach the sliding manifold in $k^\star = 30$ steps. The event parameter $\alpha = 0.05$. The simulation is run with the initial condition $x(0) = [6\ \ -2]^\top$.

The performance of the system with FOS feedback event-triggered SMC is shown in Fig. 1. The sliding trajectories reach practical QSMB in $k^\star = 30$ time steps, i.e., 3 seconds. Once the sliding mode is enforced in the system the state trajectories also remain within the practical QSMB band which is in coincidence with the theoretical developments of this paper. In the present case the magnitude of practical QSMB is calculated as 0.0523. The similar observation is also seen in the control signal. Till the sliding manifold is not reached, control is updated periodically. However, once the system trajectories enter the practical QSMB, we show that there is significant increase in inter event time $T_i = k_{i+1} - k_i$ which is defined as time interval between two consecutive triggering instants. The inter event time is increased as high as 1.5 seconds. This shows inter event time are increased sufficiently by large amount of discrete-time sampling interval.

VI. Conclusion

An event-triggering scheme for DTSM control is proposed in this paper. In this technique the triggering rule is checked only at discrete instants of time and control is updated whenever it is violated. However, the band size is increased due to this event-triggering scheme but remains bounded. The major advantage in this technique is obtained there is no Zeno execution of triggering sequences. Further, output feedback based discrete event-triggered SMC is analyzed using FOS. The outputs are sampled periodically and the event is checked for possible generation of triggering instant. In case there is no triggering, again the outputs are sampled until next discrete instant. This establishes a feasible output feedback based discrete event-triggered SMC.

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